FYJC - MATHEMATICS & STATISTICS

PAPER - I

SEQUENCES & SERIES

ARITHMETIC PROGRESSION

any sequence in the form

a , a + d , a + 2d , a + 3d , …….. is AP

where

a : first term

d : common difference

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right]$$

Q SET - 1

01. find

a) 24^{th} term of 5 , 8 , 11 , 14 , …..

ans : 74

b) 15^{th} term of 21 , 16 , 11 , 6 , …..

ans : –54

c) n^{th} term of 4 , 9 , 14 , 19 , …..

ans : 5n – 1

d) n^{th} term of 4 , $^{14}/_{3}$, $^{16}/_{3}$, 6 , …..

ans : $\frac{10 + 2n}{3}$

02. 4^{th} term of an AP is 9 and its 9^{th} term is 19 .Find the 20^{th} term

ans : 41

03. 7^{th} term of an AP is 30 and its 10^{th} term is 21 . Find the 4^{th} term

ans : 39

04. 10^{th} term and 20^{th} term of an AP are 1 and –29 respectively . Obtain 3^{rd} term

ans : 22

05. 3^{rd} term of an AP is – 11 and its 9^{th} term is – 35 . Find the n^{th} term

ans : 1 – 4n

06. insert four arithmetic means between 4 and 324

ans : 4 , 68 , 132 , 196 , 260 , 324

Q SET - 2

Find three numbers in AP such that

01. their sum is 18 and the product is 192

ans : 4 , 6 , 8

02. their sum is 24 and sum of squares is 20

ans : 6, 8 , 10

03. their sum is 27 and sum of squares is 341

ans : 2, 9 , 16

04. sum is 15 and sum of squares of two extremes is 58

ans : 3 , 5 , 7

05. sum is 12 and sum of cubes is 408

ans : 1 , 4 , 7

Q SET - 3

find four numbers in AP such that

01. their sum is 32 and sum of squares is 276

ans : 5,9,7,11

02. sum is 6 and product of whose extremes is 10 times product of means **ans :15,6,–3, –12**

03. sum of 1^{st} and last is 8 , product of 2^{nd} and 3^{rd} is 12

ans : –2 , 2 ,6,10

04. sum of 2^{nd} and 3^{rd} is 22 and product of 1^{st} & 4^{th} is 85

ans : 5 ,9, 13 , 17

05. sum is 60 and the ratio of the product of the second and third term to the product of the first and the fourth term is 3 : 2

ans : 6 , 12 , 18 , 24

Q SET - 4

01. 1 + 4 + 7 + ……………. to 22 terms

ans : 715

02. 7 + 14 + 21 + …………… to 20 terms

ans: 1470

03. − 4 , −1 , 2 , 5 , …………to 21 terms

ans : 546

04. $t_3 = 17$; $t_7 = 37$. Find S_{16}

ans : 712

05. $t_7 = 13$; $S_{14} = 203$. Find S_8

ans : 44

06. find the sum of all two digit number divisible by 7

ans : 728

07. sum of natural numbers from 1 − 200 which are divisible by 5

ans : 4100

08. find sum of all natural numbers from 100 to 300 which are exactly divisible by 13

ans : 3224

09. sum of all natural numbers from 100 − 300 which are divisible by 4

ans : 9900

Q SET - 5

How many terms are required

01. 25 + 22 + 19 + 16 + ………… = 116

ans : n = 8

02. 5 + 7 + 9 + ………………… = 480

ans : n = 20

03. 45 + 48 + 51 + ... n terms = 585

ans : n = 10

04. 50 + 46 + 42 + ………………. = 336

ans : 12, 14

05. 93 + 90 + 87 + ………………. = 975

ans : 13, 50

Q SET - 6

01. If for a sequence , $S_n = 2n^2 + 5n$, find t_n and show that the sequence is an A.P.

02. If for a sequence , $S_n = 4n^2 - 3n$, show that the sequence is an A.P.

SOLUTION - QSET 1

01.

a)

24^{th} term of 5 , 8 , 11 , 14 , …..

$a = 5$, $d = 3$
$t_{24} = a + 23d = 5 + 23(3)$

$= 5 + 69 = 74$

b)

15^{th} term of 21 , 16 , 11 , 6 , …..

$a = 21$, $d = -5$
$t_{15} = a + 14d = 21 + 15(-5)$

$= 21 - 75 = -54$

c)

n^{th} term of 4 , 9 , 14 , 19 , …..

$a = 4$, $d = 5$
$t_n = a + (n - 1)d = 4 + (n - 1)(5)$

$= 4 + 5n - 5$

$= 5n - 1$

d)

n^{th} term of 4 , $^{14}/_{3}$, $^{16}/_{3}$, 6 , …..

$a = 4$, $d = {}^{2}/_{3}$
$t_n = a + (n - 1)d = 4 + (n - 1)({}^{2}/_{3})$

$= 4 + \dfrac{2n}{3} - \dfrac{2}{3}$

$= \dfrac{10 + 2n}{3}$

02.

4^{th} term of an AP is 9 and its 9^{th} term is 19 .

Find the 20^{th} term

$$\begin{array}{lllll} t_9 = 19 & a + 8d & = & 19 \\ t_4 = 9 & a + 3d & = & 9 \\ \hline & 5d & = & 10 \\ & d & = & 2 \\ & a & = & 3 \end{array}$$

$t_{20} = a + 19d = 3 + 19(2)$

$= 3 + 38 = 41$

03.

7^{th} term of an AP is 30 and its 10^{th} term is 21 .

Find the 4^{th} term

$$\begin{array}{lllll} t_{10} = 21 & a + 9d & = & 21 \\ t_7 = 30 & a + 6d & = & 30 \\ \hline & 3d & = & -9 \\ & d & = & -3 \\ & a & = & 48 \end{array}$$

$t_4 = a + 3d = 48 + 3(-3)$

$= 48 - 9 = 39$

04.

10^{th} term and 20^{th} term of an AP are 1 & −29 respectively . Obtain 3^{rd} term

$$\begin{array}{lllll} t_{20} = -29 & a + 19d & = & -29 \\ t_{10} = 1 & a + 9d & = & 1 \\ \hline & 10d & = & -30 \\ & d & = & -3 \\ & a & = & 28 \end{array}$$

$t_3 = a + 2d = 28 + 2(-3)$

$= 28 - 6 = 22$

05.

3^{rd} term of an AP is − 11 & its 9^{th} term is − 35 .

Find the n^{th} term

$$\begin{array}{lllll} t_9 = -35 & a + 8d & = & -35 \\ t_3 = -11 & a + 2d & = & -11 \\ - & - & & + \\ \hline & 6d & = & -24 \\ & d & = & -4 \\ & a & = & -3 \end{array}$$

$t_n = a + (n-1)d = -3 + (n - 1)(-4)$

$= -3 - 4n + 4$

$= 1 - 4n$

06.

insert four arithmetic means between 4 and 324

$t_1 = 4 \quad a = 4$

$t_6 = 324 \quad a + 5d = 324$

$4 + 5d = 324$

$5d = 320$

$d = 64$

$t_2 = a + d = 4 + 64 = 68$

$t_3 = a + 2d = 4 + 2(64) = 4 + 128 = 132$

$t_4 = a + 3d = 4 + 3(64) = 4 + 192 = 196$

$t_5 = a + 4d = 4 + 4(64) = 4 + 256 = 260$

SOLUTION - QSET 2

Find three numbers in AP such that

01. their sum is 18 and the product is 192

SOLUTION

let 3 nos in AP : $a - d , \quad a \quad , \quad a + d$

sum $= 18$

$3a = 18 \qquad \therefore a = 6$

product $= 192$

$(a - d).a.(a + d) = 192$

$(6 - d).6.(6 + d) = 192$

$(6 - d).(6 + d) = 32$

$36 - d^2 = 32$

$36 - 32 = d^2$

$d^2 = 4 \qquad \therefore d = \pm 2$

$a = 6 , d = 2$

nos are : $a - d , a , a + d$

$6 - 2 , 6 , 6 + 2$

$4 , 6 , 8$

$a = 6 , d = -2$

nos are : $a - d , a , a + d$

$6 + 2 , 6 , 6 - 2$

$8 , 6 , 4$

02. their sum is 24 & sum of squares is 200

SOLUTION

let 3 nos in AP : $a - d , \quad a \quad , \quad a + d$

sum $= 24$

$3a = 24 \qquad \therefore a = 8$

sum of squares $= 200$

$(a - d)^2 + a^2 + (a + d)^2 = 200$

$a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 200$

$3a^2 + 2d^2 = 200$

$3(8)^2 + 2d^2 = 200$

$3(64) + 2d^2 = 200$

$192 + 2d^2 = 200$

$2d^2 = 8$

$d^2 = 4 \qquad \therefore d = \pm 2$

$a = 8 , d = 2$

nos are : $a - d , a , a + d$

$8 - 2 , 8 , 8 + 2$

$6 , 8 , 10$

$a = 8 , d = -2$

nos are : $a - d , a , a + d$

$8 + 2 , 8 , 8 - 2$

$10 , 8 , 6$

03. their sum is 27 & sum of squares is 341

SOLUTION

let 3 nos in AP : $a - d , \quad a \quad , \quad a + d$

sum $= 27$

$3a = 27 \qquad \therefore a = 9$

sum of squares $= 341$

$(a - d)^2 + a^2 + (a + d)^2 = 341$

$a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 341$

$3a^2 + 2d^2 = 341$

$3(9)^2 + 2d^2 = 341$

$3(81) + 2d^2 = 341$

$243 + 2d^2 = 341$

$2d^2 = 98$

$d^2 = 49 \quad \therefore d = \pm 7$

$a = 9, d = 7$

nos are : $a - d, a, a + d$

$9 - 7, 9, 9 + 7$

$2, 9, 16$

$a = 9, d = -7$

nos are : $a - d, a, a + d$

$9 + 7, 9, 9 - 7$

$16, 9, 2$

04.

sum is 15 & sum of squares of extremes is 58

SOLUTION

let 3 nos in AP : $a - d, \ a, \ a + d$

sum $= 15$

$3a = 15 \quad \therefore a = 5$

sum of two extremes $= 58$

$(a - d)^2 + (a + d)^2 = 58$

$a^2 - 2ad + d^2 + a^2 + 2ad + d^2 = 58$

$2a^2 + 2d^2 = 58$

$2(5)^2 + 2d^2 = 58$

$50 + 2d^2 = 58$

$2d^2 = 8$

$d^2 = 4 \quad \therefore d = \pm 2$

$a = 5, d = 2$

nos are : $a - d, a, a + d$

$5 - 2, 5, 5 + 2$

$3, 5, 7$

$a = 5, d = -2$

nos are : $a - d, a, a + d$

$5 + 2, 5, 5 - 2$

$7, 5, 3$

05. sum is 12 and sum of cubes is 408

SOLUTION

let 3 nos in AP : $a - d, \ a, \ a + d$

sum $= 12$

$3a = 12 \quad \therefore a = 4$

sum of cubes $= 408$

$(a - d)^3 + a^3 + (a + d)^3 = 408$

$$\begin{array}{rl} & a^3 - 3a^2d + 3ad^2 + d^3 \\ + & a^3 \qquad\qquad\qquad\qquad = 408 \\ + & a^3 + 3a^2d + 3ad^2 - d^3 \\ \hline & 3a^3 \qquad + 6ad^2 \qquad = 408 \end{array}$$

$\div 3$

$a3 + 2ad^2 = 136$

$4^3 + 2(4)d^2 = 136$

$64 + 8d^2 = 136$

$8d^2 = 72$

$d^2 = 9 \quad \therefore d = \pm 3$

$a = 4, d = 3$

nos are : $a - d, a, a + d$

$4 - 3, 4, 4 + 3$

$1, 4, 7$

$a = 4, d = -3$

nos are : $a - d, a, a + d$

$4 + 3, 4, 4 - 3$

$7, 4, 1$

SOLUTION - QSET 3

find four numbers in AP such that

01.

sum of 1st & last is 8 , product of 2nd & 3rd is 12

SOLUTION

let 4 nos in AP : $a - 3d$, $a - d$, $a + d$, $a + 3d$

sum of 1st & last is 8

$2a = 8 \quad \therefore a = 4$

product of 2nd & 3rd is 12

$(a - d)(a + d) = 12$

$a^2 - d^2 = 12$

$16 - d^2 = 12$

$d^2 = 4 \quad \therefore d = \pm 2$

$a = 4$, $d = 2$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$4 - 6$, $4 - 2$, $4 + 2$, $4 + 6$

-2 , 2 , 6 , 10

$a = 4$, $d = -2$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$4 + 6$, $4 + 2$, $4 - 2$, $4 - 6$

10 , 6 , 2 , -2

02.

sum of 2nd and 3rd is 22 and product of 1st & 4th is 85

SOLUTION

let 4 nos in AP : $a - 3d$, $a - d$, $a + d$, $a + 3d$

sum of 2nd and 3rd is 22

$2a = 22 \quad \therefore a = 11$

product of 1st & 4th is 85

$(a - 3d)(a + 3d) = 85$

$a^2 - 9d^2 = 85$

$121 - 9d^2 = 85$

$121 - 85 = 9d^2$

$9d^2 = 36$

$d^2 = 4 \quad \therefore d = \pm 2$

$a = 11$, $d = 2$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$11 - 6$, $11 - 2$, $11 + 2$, $11 + 6$

5 , 9 , 13 , 17

$a = 11$, $d = -2$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$11 + 6$, $11 + 2$, $11 - 2$, $11 - 6$

17 , 13 , 9 , 5

03.

their sum is 32 and sum of squares is 276

SOLUTION

let 4 nos in AP : $a - 3d$, $a - d$, $a + d$, $a + 3d$

sum $= 32$

$4a = 32 \quad \therefore a = 8$

sum of squares $= 276$

$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 276$

$$\begin{array}{rl} & a^2 - 6ad + 9d^2 \\ + & a^2 - 2ad + d^2 \\ + & a^2 + 2ad + d^2 \\ + & a^2 + 6ad + 9d^2 = 276 \\ \hline & 4a^2 + 20d^2 = 276 \end{array}$$

$4(8)^2 + 20d^2 = 276$

$4(64) + 20d^2 = 276$

$256 + 20d^2 = 276$

$20d^2 = 20$

$d^2 = 1 \quad \therefore d = \pm 1$

$a = 8$, $d = 1$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$8 - 3$, $8 - 1$, $8 + 1$, $8 + 3$

5 , 7 , 9 , 11

$a = 8$, $d = -1$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$8 + 3$, $8 + 1$, $8 - 1$, $8 - 3$

11 , 9 , 7 , 5

04.

sum is 6 and product of whose extremes is 10 times product of means

SOLUTION

let 4 nos in AP : $a - 3d$, $a - d$, $a + d$, $a + 3d$

sum = 6

$4a = 6 \qquad \therefore a = 3/2$

product of whose extremes is 10 times product of means

$(a - 3d).(a + 3d) = 10(a - d)(a + d)$

$a^2 - 9d^2 = 10(a^2 - d^2)$

$a^2 - 9d^2 = 10a^2 - 10d^2$

$d^2 = 9a^2$

$d^2 = 9(3/2)^2$

$d^2 = 9.\frac{9}{4}$

$d^2 = \frac{81}{4} \qquad \therefore d = \pm\frac{9}{2}$

$a = 3/2$, $d = 9/2$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$\frac{3}{2} - \frac{27}{2} , \frac{3}{2} - \frac{9}{2} , \frac{3}{2} + \frac{9}{2} , \frac{3}{2} + \frac{27}{2}$

$\frac{-24}{2} , \frac{-6}{2} , \frac{12}{2} , \frac{30}{2}$

$-12 , -3 , 6 , 15$

$a = 3/2$, $d = -9/2$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$\frac{3}{2} + \frac{27}{2} , \frac{3}{2} + \frac{9}{2} , \frac{3}{2} - \frac{9}{2} , \frac{3}{2} - \frac{27}{2}$

$\frac{30}{2} , \frac{12}{2} , \frac{-6}{2} , \frac{-24}{2}$

$15 , 6 , -3 , -12$

05. sum is 60 and the ratio of the product of the second and third term to the product of the first and the fourth term is 3 : 2

SOLUTION

let 4 nos in AP : $a - 3d$, $a - d$, $a + d$, $a + 3d$

sum = 60

$4a = 60 \qquad \therefore a = 15$

ratio of the product of the second and third term to the product of the first and the fourth term is 3 : 2

$\frac{(a - d)(a + d)}{(a - 3d)(a + 3d)} = \frac{3}{2}$

$\frac{a^2 - d^2}{a^2 - 9d^2} = \frac{3}{2}$

$2a^2 - 2d^2 = 3a^2 - 27d^2$

$25d^2 = a^2$

$25d^2 = 225$

$d^2 = 9 \qquad \therefore d = \pm 3$

$a = 15$, $d = 3$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$15 - 9 , 15 - 3 , 15 + 3 , 15 + 9$

$6 , 12 , 18 , 14$

$a = 15$, $d = -3$

nos are : $a - 3d$, $a - d$, $a + d$, $a + 3d$

$15 + 9 , 15 + 3 , 15 - 3 , 15 - 9$

$24 , 18 , 12 , 6$

SOLUTION - QSET 4

01. $1 + 4 + 7 + \ldots\ldots\ldots\ldots.$ to 22 terms

SOLUTION

$a = 1, \quad d = 3, n = 22$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{22} = \frac{22}{2}\left[2(1) + (22-1)3\right]$

$= 11\left[2 + 21(3)\right]$

$= 11(2 + 63)$

$= 11(65) \quad = 715$

02. $7 + 14 + 21 + \ldots\ldots\ldots\ldots$ to 20 terms

SOLUTION

$a = 7, \quad d = 7 \quad , \quad n = 20$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{20} = \frac{20}{2}\left[2(7) + (20-1)7\right]$

$= 10\left[14 + 19(7)\right]$

$= 10(14 + 133)$

$= 11(147) \quad = 1470$

03. $-4, -1, 2, 5, \ldots\ldots\ldots$ to 21 terms

SOLUTION

$a = -4 \quad , \quad d = 3 \quad , \quad n = 21$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{21} = \frac{21}{2}\left[2(-4) + (21-1)3\right]$

$= \frac{21}{2}\left[-8 + 20(3)\right]$

$= \frac{21}{2}(-8 + 60)$

$= \frac{21}{2}(52)$

$= 21(26) \quad = 546$

04. $t_3 = 17 \;;\; t_7 = 37$. Find S_{16}

$t_7 = 37 \qquad a + 6d = 37$

$t_3 = 17 \qquad a + 2d = 17$

$4d = 20$

$d = 5$

subs in 1 $\qquad a = 7$

$a = 7, \quad d = 5, \quad n = 16$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{16} = \frac{16}{2}\left[2(7) + (16-1)5\right]$

$= 8\left[14 + 15(5)\right]$

$= 8(14 + 75)$

$= 8(89)$

$= 712$

05. $t_7 = 13 \;;\; S_{14} = 203$. Find S_8

SOLUTION

$t_7 = 13 \qquad a + 6d = 13 \ldots (1)$

$S_{14} = 203$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S14 = \frac{14}{2}(2a + 13d)$

$203 = 7(2a + 13d)$

$2a + 13d = 29 \ldots\ldots \quad (2)$

Solving (1) & (2)

$a = -5 \quad , \quad d = 3$

Now

$a = -5 \quad , \quad d = 3 \quad , \quad n = 8$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_8 = \frac{8}{2}\left[2(-5) + (8-1)3\right]$

$= 4\left[-10 + 7(3)\right]$

$= 4(-10 + 21)$

$= 4(11)$

$= 44$

06. find the sum of all two digit number divisible by 7

SOLUTION

$14 + 21 + 28 + \ldots\ldots. + 98$

$a = 14, d = 7$

$t_n = a + (n-1)d$

$98 = 14 + (n-1)7$

$84 = (n-1)7$

$12 = n - 1$

$n = 13$

Now

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{13} = \frac{13}{2}\left[2(14) + (13-1)7\right]$$

$$= \frac{13}{2}\left[28 + 12(7)\right]$$

$$= \frac{13}{2}(28 + 84)$$

$$= \frac{13}{2}(112)$$

$$= 13(56)$$

$$= 728$$

07. sum of natural numbers from 1 – 200 which are divisible by 5

SOLUTION

5 + 10 + ……. + 200

$a = 5$, $d = 5$

$t_n = a + (n-1)d$

$200 = 5 + (n-1)5$

$195 = (n-1)5$

$39 = n - 1$

$n = 40$

Now

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(5) + (40-1)5\right]$$

$$= 20\left[10 + 39(5)\right]$$

$$= 20(10 + 195)$$

$$= 20(205)$$

$$= 4100$$

08. find sum of all natural numbers from 100 to 300 which are exactly divisible by 13

SOLUTION

104 + 117 + ……. + 299

$a = 104$, $d = 13$

$t_n = a + (n-1)d$

$299 = 104 + (n-1)13$

$195 = (n-1)13$

$15 = n - 1$

$n = 16$

$$S_{16} = \frac{16}{2}\left[2(104) + (16-1)13\right]$$

$$= 8\left[208 + 15(13)\right]$$

$$= 8(208 + 195)$$

$$= 8(403)$$

$$= 3224$$

09. sum of all natural numbers from 100 – 300 which are divisible by 4

SOLUTION

100 + 104 + ……. + 300

$a = 104$, $d = 4$

$t_n = a + (n-1)d$

$300 = 104 + (n-1)4$

$196 = (n-1)4$

$49 = n - 1$

$n = 50$

Now

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{50} = \frac{50}{2}\left[2(100) + (50-1)4\right]$$

$$= 25\left[200 + 49(4)\right]$$

$$= 25(200 + 196)$$

$$= 25(396)$$

$$= 9900$$

SOLUTION - QSET 5

How many terms are required

01. $25 + 22 + 19 + 16 + \ldots\ldots\ldots = 116$

SOLUTION

$a = 25, \quad d = -3, \quad S_n = 116$

Now

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$116 = \frac{n}{2}\left(2(25) + (n-1)(-3)\right)$

$232 = n\left(50 - 3n + 3\right)$

$232 = n\left(53 - 3n\right)$

$232 = 53n - 3n^2$

$3n^2 - 53n + 232 = 0$

$3n^2 - 24n - 29n + 232 = 0$

$3n(n-8) - 29(n-8) = 0$

$(3n - 29)(n - 8) = 0$

$n \neq 29/3 \;;\; n = 8$

02. $5 + 7 + 9 + \ldots\ldots\ldots\ldots\ldots\ldots = 480$

SOLUTION

$a = 5, \quad d = 2, \quad S_n = 480$

Now

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$480 = \frac{n}{2}\left(2(5) + (n-1)(2)\right)$

$960 = n\left(10 + 2n - 2\right)$

$960 = n\left(8 + 2n\right)$

$960 = 8n + 2n^2$

$2n^2 + 8n - 960 = 0$

$n^2 + 4n - 480 = 0$

$n^2 + 24n - 20n - 480 = 0$

$n(n + 24) - 20(n + 24) = 0$

$(n - 20)(n + 24) = 0$

$n = 20 \;;\; n \neq 24$

03. $45 + 48 + 51 + \ldots\ldots$ n terms $= 585$

SOLUTION

$a = 45, \quad d = 3, \quad S_n = 585$

Now

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$585 = \frac{n}{2}\left(2(45) + (n-1)(3)\right)$

$1170 = n\left(90 + 3n - 3\right)$

$1170 = n\left(87 + 3n\right)$

$1170 = 87n + 3n^2$

$3n^2 + 87n - 1170 = 0$

$n^2 + 29n - 390 = 0$

$n^2 + 39n - 10n - 390 = 0$

$n(n + 39) - 10(n + 39) = 0$

$(n - 10)(n + 29) = 0$

$n = 10 \;;\; n \neq 29$

04. $50 + 46 + 42 + \ldots\ldots\ldots\ldots\ldots\ldots. = 336$

SOLUTION

$a = 50, \quad d = -4, \quad S_n = 336$

Now

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$336 = \frac{n}{2}\left(2(50) + (n-1)(-4)\right)$

$672 = n\left(100 - 4n + 4\right)$

$672 = n\left(104 - 4n\right)$

$672 = 104n - 4n^2$

$4n^2 - 104n + 672 = 0$

$n^2 - 26n + 168 = 0$

$n^2 - 14n - 12n + 168 = 0$

$n(n - 14) - 12(n - 14) = 0$

$(n - 12)(n - 14) = 0$

$n = 12 \;;\; n = 14$

05. $93 + 90 + 87 + \ldots\ldots\ldots\ldots = 975$

SOLUTION

$a = 93$, $d = -3$, $Sn = 975$

Now

$$Sn = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$975 = \frac{n}{2}\left(2(93) + (n-1)(-3)\right)$$

$$1950 = n\left(186 - 3n + 3\right)$$

$$1950 = n\left(189 - 3n\right)$$

$$1950 = 189n - 3n^2$$

$$3n^2 - 189n + 1950 = 0$$

$$n^2 - 63n + 650 = 0$$

$$n^2 - 50n - 13n + 650 = 0$$

$$n(n-50) - 13(n-50) = 0$$

$$(n-13)(n-50) = 0$$

$$n = 13 \quad ; \quad n = 50$$

SOLUTION - QSET 6

01.

If for a sequence , $Sn = 2n^2 + 5n$, find tn and show that the sequence is an A.P.

SOLUTION

$$t_n = S_n - S_{n-1}$$

$$= 2n^2 + 5n - \left(2(n-1)^2 + 5(n-1)\right)$$

$$= 2n^2 + 5n - \left(2(n^2 - 2n + 1) + 5n - 5\right)$$

$$= 2n^2 + 5n - \left(2n^2 - 4n + 2 + 5n - 5\right)$$

$$= 2n^2 + 5n - \left(2n^2 + n - 3\right)$$

$$= 2n^2 + 5n - 2n^2 - n + 3$$

$$= 4n + 3$$

$$t_{n-1} = 4(n-1) + 3$$

$$= 4n - 4 + 3$$

$$= 4n - 1$$

$$t_n - t_{n-1} = 4n + 3 - (4n - 1)$$

$$= 4n + 3 - 4n + 1$$

$= 4 =$ constant Hence AP

02.

If for a sequence , $Sn = 4n^2 - 3n$, show that the sequence is an A.P.

SOLUTION

$$t_n = S_n - S_{n-1}$$

$$= 4n^2 - 3n - \left(4(n-1)^2 - 3(n-1)\right)$$

$$= 4n^2 - 3n - \left(4(n^2 - 2n + 1) - 3n + 3\right)$$

$$= 4n^2 - 3n - \left(4n^2 - 8n + 4 - 3n + 3\right)$$

$$= 4n^2 - 3n - \left(4n^2 - 11n + 7\right)$$

$$= 4n^2 - 3n - 4n^2 + 11n - 7$$

$$= 8n - 7$$

$$t_{n-1} = 8(n-1) - 7$$

$$= 8n - 8 - 7$$

$$= 8n - 15$$

$$t_n - t_{n-1} = 8n - 7 - (8n - 15)$$

$$= 8n - 7 - 8n + 15$$

$$= 8$$

$=$ constant Hence AP

GEOMETRIC PROGRESSION

Q SET - 1

Q1.

Find tn for the following GP

01. 3 , 15 , 75 , 375 , ...

    ans : $3(5)^{n-1}$

02. 1 , −4 , 16 , −64 , ...

    ans : $(-4)^{n-1}$

03. $1 , -\frac{3}{2} , \frac{9}{4} , -\frac{27}{8}$, ...

    ans : $(-\frac{3}{2})^{n-1}$

04. $\sqrt{3} , \frac{1}{\sqrt{3}} , \frac{1}{3\sqrt{3}} , \frac{1}{9\sqrt{3}}$, ...

    ans : $\left(\frac{1}{3}\right)^{n-3/2}$

Q2.

01. $a = 7$, $r = 1/3$, find t_6

    ans : 7/243

02. $a = 5$, $r = -2$, find t_5

    ans : 80

03. $a = \frac{2}{3}$, $t_6 = 162$ find r

    ans : 3

04. $t_8 = 640$, $r = 2$ find a

    ans : 5

05. $a = 5$, $t_6 = \frac{1}{625}$ find r , t_{10}

    ans : $\frac{1}{5}$, $\left(\frac{1}{5}\right)^8$

07. If in a GP , $t_3 = 18$, $t_6 = 486$. Find t_9

    ans : 13122

08. insert two numbers between 3 & 81 such that the resulting sequence is a G.P.

    ans : 9 , 27

09. insert two numbers between 3 & 24 such that the resulting sequence is a G.P.

    ans : 6 , 12

10. insert 2 numbers between −40 & −5 such that the resulting sequence is a G.P.

    ans : −20 , −10

11. insert 5 geometric means between 3 & 192 such that the resulting sequence is a G.P.

    ans : 6 , 12 , 24 , 48 , 96

Q SET - 2

Find THREE numbers in GP such that ;

01. sum is 35 and product is 1000

    ans : 5 , 10 , 20

02. sum is 26 and product is 216

    ans : 2 , 6 ,18

03. sum is 28 and product is 512

    ans : 4 , 8 ,16

04. sum is $4\frac{1}{3}$ and product is 1

    ans : 3 , 1 , $\frac{1}{3}$

05. product is 216, sum of 1^{st} & 3^{rd} is 20

    ans : 2 , 6 , 18

06. sum is 21 & sum of squares is 189

    ans : 3 , 6 , 12

07. sum is $\frac{13}{3}$ & sum of squares is $\frac{91}{9}$

    ans : 3 , 1 , $\frac{1}{3}$

08. sum is 26 & sum of squares is 364

    ans : 2 , 6 , 18

Find FIVE numbers in GP such that

09. product = 1024 and sum of second and fourth is 10

ans : 1 , 2 , 4 , 8 , 16

10. product = 32 and product of fourth and fifth number is 108

ans : $\frac{2}{9}$, $\frac{2}{3}$, 2 , 6 , 18

Find FOUR numbers in GP such that

11. product is 1 and sum of middle two nos is $^{10}/_{3}$

ans : $\frac{1}{27}$, $\frac{1}{3}$, 3 , 27

12. product is 729 and sum of second & third nos is 12

ans : 1 , 3 , 9 , 27

Q SET - 3

01. in G.P. $t_3 = 36$; $t_6 = 972$. Find S_4

ans : 160

02. in G.P. $t_3 = 18$; $t_6 = 486$. Find S_5

ans : 242

03. in G.P. $t_4 = 24$; $t_9 = 768$. Find S_8

ans : 765

04. in G.P. $S_3 = 125$; $S_6 = 152$. Find r

ans : 3/5

05. in G.P. $S_4 = 81$; $S_8 = 97$. Find r

ans : ± 2/3

06. if 1 + 4 + 16 + 64 + ... n terms = 5461 . Find n

ans : 7

07. $\sqrt{3} + 3 + 3\sqrt{3} + \ldots\ldots = 39 + 13\sqrt{3}$. Find n

ans : 6

08. $\sqrt{3} + 3 + 3\sqrt{3} + \ldots\ldots = 120 + 40\sqrt{3}$. Find n

ans : 8

Q SET - 4

Q1. Find Sn

01. 9 + 99 + 999 + ...

ans : $\frac{10\,(10^n - 1\,) - 9n}{10}$

02. 7 + 77 + 777 + ...

ans : $\frac{7}{81}\left[10(10^n - 1) - 9n\right]$

03. 8 + 88 + 888 + ...

ans : $\frac{8}{81}\left[10(10^n - 1) - 9n\right]$

Q2. Find Sn

01. 0.9 + 0.99 + 0.999 + ...

ans : $\frac{9n - 1 + 0.1^n}{9}$

02. 0.8 + 0.88 + 0.888 + ...

ans : $\frac{8}{81}$ (9n –1 + 0.1^n)

03. 0.3 + 0.33 + 0.333 + ...

ans : $\frac{1}{27}$ (9n –1 + 0.1^n)

Q3. Find tn

01. 0.3 , 0.33 , 0.333 + ...

ans : $\frac{1}{3}$ $(1 - 0.1^n)$

02. 0.4 , 0.44 , 0.444 + ...

ans : $\frac{4}{9}$ $(1 - 0.1^n)$

03. 0.7 , 0.77 , 0.777 + ...

ans : $\frac{7}{9}$ $(1 - 0.1^n)$

Q SET - 5

Q1.

01. if for a sequence ; $t_n = \dfrac{2^{n-2}}{5^{n-3}}$;

show that sequence is a G.P. Find its first term and the common ratio

02. if for a sequence ; $t_n = \dfrac{4^{n-3}}{5^{n-2}}$;

show that sequence is a G.P. Find its first term and the common ratio

03. if for a sequence ; $t_n = \dfrac{7^{n+3}}{5^{n+2}}$;

show that sequence is a G.P. Find its first term and the common ratio

Q2.

01. for a sequence ; if $S_n = 5(2^n - 1)$; find t_n and show that the sequence is a G.P.

02. for a sequence ; if $S_n = 3(2^n - 1)$; find t_n and show that the sequence is a G.P.

03. for a sequence ; if $S_n = \dfrac{4^n - 3^n}{3^n}$;

find t_n and show that the sequence is a G.P

SOLUTION - QSET 1

Q1.

Find tn for the following GP

01. $3 , 15 , 75 , 375 , \ldots\ldots$

$a = 3 \; ; \; r = 5$

$tn = ar^{n-1}$

$= 3(5)^{n-1}$

02. $1 , -4 , 16 , -64 , \ldots\ldots$

$a = 1 \; ; \; r = -4$

$tn = ar^{n-1}$

$= 1(-4)^{n-1}$

$= (-4)^{n-1}$

03. $1 , -{}^{3}/_{2} , {}^{9}/_{4} , -{}^{27}/_{8} , \ldots\ldots$

$a = 1 \; ; \; r = -{}^{3}/_{2}$

$tn = ar^{n-1}$

$= 1(-{}^{3}/_{2})^{n-1}$

$= (-{}^{3}/_{2})^{n-1}$

04. $\sqrt{3} , \frac{1}{\sqrt{3}} , \frac{1}{3\sqrt{3}} , \frac{1}{9\sqrt{3}} , \ldots$

$a = \sqrt{3} , \quad r = \frac{1}{3}$

$tn = ar^{n-1}$

$= \sqrt{3} . \left(\frac{1}{3}\right)^{n-1}$

$= \frac{3^{1/2}}{3^{n-1}}$

$= \frac{1}{3^{n-1-1/2}}$

$= \frac{1}{3^{n-3/2}}$

$= \left(\frac{1}{3}\right)^{n-3/2}$

Q2.

01. $a = 7 , \quad r = 1/3$, find t_6

$t_6 = ar^5$

$= 7\left(\frac{1}{3}\right)^5 = \frac{7}{243}$

02. $a = 5 , \quad r = -2$, find t_5

$t_5 = ar^4$

$= 5(-2)^4$

$= 5(16) = 80$

03. $a = {}^{2}/_{3}$, $t_6 = 162$ find r

$t_6 = ar^5$

$162 = \frac{2}{3} r^5$

$r^5 = 243$

$r^5 = 3^5 \qquad r = 3$

04. $t_8 = 640$, $r = 2$ find a

$t_8 = ar^7$

$640 = a\,2^7$

$640 = a(128) \qquad a = 5$

05. $a = 5$, $t_6 = \frac{1}{625}$ find r

$t_6 = ar^5$

$\frac{1}{625} = 5r^5$

$r^5 = \frac{1}{3125}$

$r^5 = \left(\frac{1}{5}\right)^5 \qquad r = \frac{1}{5}$

$t_{10} = ar^9$

$= 5\left(\frac{1}{5}\right)^9 = \left(\frac{1}{5}\right)^8$

07. If in a GP , $t_3 = 18$, $t_6 = 486$. Find t_9

$t_3 = 18 \quad \therefore \; ar^2 = 18$

$t_3 = 486 \quad \therefore \; ar^5 = 486$

$\frac{ar^5}{ar^2} = \frac{486}{18}$

$r^3 = 27 \qquad \therefore \; r = 3$

subs in

$ar^2 = 18 \qquad \therefore \; a = 2$

$t_9 = ar^8 = 2(3)^8 = 13122$

08. insert two numbers between 3 & 81 such that the resulting sequence is a G.P.

$t_1 = 3 \quad \therefore a = 3$

$t_4 = 81 \quad \therefore ar^3 = 81$

$$\frac{ar^3}{a} = \frac{81}{3}$$

$$r^3 = 27$$

$$r = 3$$

$a = 3 \ , \ r = 3$

$t_2 = ar = 9$

$t_3 = ar^2 = 3(9) = 27$

09. insert two numbers between 3 & 24 such that the resulting sequence is a G.P.

$t_1 = 3 \quad \therefore a = 3$

$t_4 = 24 \quad \therefore ar^3 = 24$

$$\frac{ar^3}{a} = \frac{24}{3}$$

$$r^3 = 8$$

$$r = 2$$

$a = 3 \ , \ r = 2$

$t_2 = ar = 6$

$t_3 = ar^2 = 3(4) = 12$

10. insert 2 numbers between −40 & −5 such that the resulting sequence is a G.P.

$t_1 = -40 \quad \therefore a = -40$

$t_4 = -5 \quad \therefore ar^3 = -5$

$$\frac{ar^3}{a} = \frac{-5}{-40}$$

$$r^3 = \frac{1}{8}$$

$$r = \frac{1}{2}$$

$a = -40 \ , \ r = {}^1/_2$

$t_2 = ar = -20$

$t_3 = ar^2 = -40 \times {}^1/_4 = -10$

11. insert 5 geometric means between 3 & 192

$3 , t_2 , t_3 , t_4 , t_5 , t_6 , 192$

$t_1 = 3 \quad \therefore a = 3$

$t_7 = 192 \quad \therefore ar^6 = 192$

$$\frac{ar^6}{a} = \frac{192}{3}$$

$$r^6 = 64$$

$$r = 2$$

$a = 3 \ , \ r = 2$

$t_2 = ar = 6$

$t_3 = ar^2 = 3(4) = 12$

$t_4 = ar^3 = 3(8) = 24$

$t_5 = ar^4 = 3(16) = 48$

$t_6 = ar^5 = 3(32) = 96$

$\therefore$ 5 GM's are 6 , 12 , 24 , 48 , 96

SOLUTION - QSET 2

01. sum is 35 and product is 1000

let 3 nos. in GP be $\frac{a}{r}$, a , ar

Product $= 1000$

$$\frac{a}{r} \cdot a \cdot ar = 1000$$

$$a^3 = 1000$$

$$a = 10$$

sum $= 35$

$$\frac{a}{r} + a + ar = 35$$

$$a\left(\frac{1}{r} + 1 + r\right) = 35$$

$$\frac{1}{r} + 1 + r = \frac{35}{a}$$

$$\frac{1}{r} + 1 + r = \frac{35}{10}$$

$$\frac{1}{r} + r = \frac{7}{2} - 1$$

$$\frac{1 + r^2}{r} = \frac{5}{2}$$

$$2 + 2r^2 = 5r$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r - 2) - 1(r - 2) = 0$$

$$(2r - 1)(r - 2) = 0$$

$$r = \frac{1}{2} \; ; \; r = 2$$

Hence

$a = 10$, $r = \frac{1}{2}$	$a = 10$, $r = 2$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{20 , 10 , 5}$	$\boxed{5 , 10 , 20}$

02. sum is 26 and product is 216

let 3 nos. in GP be $\frac{a}{r}$, a , ar

Product $= 216$

$$\frac{a}{r} \cdot a \cdot ar = 216$$

$$a^3 = 216$$

$$a = 6$$

sum $= 26$

$$\frac{a}{r} + a + ar = 26$$

$$a\left(\frac{1}{r} + 1 + r\right) = 26$$

$$\frac{1}{r} + 1 + r = \frac{26}{a}$$

$$\frac{1}{r} + 1 + r = \frac{26}{6}$$

$$\frac{1}{r} + r = \frac{13}{3} - 1$$

$$\frac{1 + r^2}{r} = \frac{10}{3}$$

$$3 + 3r^2 = 10r$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - 9r - r + 3 = 0$$

$$3r(r - 3) - 1(r - 3) = 0$$

$$(3r - 1)(r - 3) = 0$$

$$r = \frac{1}{3} \; ; \; r = 3$$

Hence

$a = 6$, $r = 3$	$a = 6$, $r = \frac{1}{3}$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{2 , 6 , 18}$	$\boxed{18, 6 , 2}$

03. sum is 28 and product is 512

HINT : REFER SOLN 02 ,
$a = 8$, $r = 2$, $1/2$

04. sum is $4\frac{1}{3}$ and product is 1

let 3 nos. in GP be $\frac{a}{r}$, a , ar

Product = 1

$$\frac{a}{r} \cdot a \cdot ar = 1$$

$$a^3 = 1$$

$$a = 1$$

sum = 13/3

$$\frac{a}{r} + a + ar = \frac{13}{3}$$

$$a\left(\frac{1}{r} + 1 + r\right) = \frac{13}{3}$$

$$\frac{1}{r} + 1 + r = \frac{13}{3}$$

$$\frac{1}{r} + r = \frac{13}{3} - 1$$

$$\frac{1 + r^2}{r} = \frac{10}{3}$$

$$3 + 3r^2 = 10r$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - 9r - r + 3 = 0$$

$$3r(r-3) - 1(r-3) = 0$$

$$(3r-1)(r-3) = 0$$

$$r = \frac{1}{3} \;;\; r = 3$$

Hence

$a = 1$, $r = 3$	$a = 1$, $r = \frac{1}{3}$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{\frac{1}{3} , 1 , 3}$	$\boxed{3 , 1 , \frac{1}{3}}$

05. product is 216, sum of 1st & 3rd is 20

let 3 nos. in GP be $\frac{a}{r}$, a , ar

Product = 216

$$\frac{a}{r} \cdot a \cdot ar = 216$$

$$a^3 = 216$$

$$a = 6$$

sum of 1st & 3rd = 20

$$\frac{a}{r} + ar = 20$$

$$a\left(\frac{1}{r} + r\right) = 20$$

$$\frac{1}{r} + r = \frac{20}{a}$$

$$\frac{1}{r} + r = \frac{20}{6}$$

$$\frac{1 + r^2}{r} = \frac{10}{3}$$

$$3 + 3r^2 = 10r$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - 9r - r + 3 = 0$$

$$3r(r-3) - 1(r-3) = 0$$

$$(3r-1)(r-3) = 0$$

$$r = \frac{1}{3} \;;\; r = 3$$

Hence

$a = 6$, $r = 3$	$a = 6$, $r = \frac{1}{3}$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{2 , 6 , 18}$	$\boxed{18 , 6 , 2}$

06. sum is 21 & sum of squares is 189

let 3 nos. in GP be $\frac{a}{r}$, a , ar

sum = 21

$$\frac{a}{r} + a + ar = 21$$

$$a\left(\frac{1}{r} + 1 + r\right) = 21$$

$$\frac{1}{r} + 1 + r = \frac{21}{a}$$

$$\frac{1}{r} + r = \frac{21}{a} - 1$$

sum of squares is 189

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = 189$$

$$a^2\left(\frac{1}{r^2} + 1 + r^2\right) = 189$$

$$\frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$$

$$\frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1$$

NOW

$$\frac{1}{r^2} + r^2 = \left(\frac{1}{r} + r\right)^2 - 2$$

$$\frac{189}{a^2} - 1 = \left(\frac{21}{a} - 1\right)^2 - 2$$

$$\frac{189}{a^2} - 1 = \frac{441}{a^2} - \frac{42}{a} + 1 - 2$$

$$\frac{189}{a^2} = \frac{441}{a^2} - \frac{42}{a}$$

$$\frac{189}{a^2} = \frac{441 - 42a}{a^2}$$

$$189 = 441 - 42a$$

$$42a = 252$$

$$a = 6$$

subs $a = 6$ in

$$\frac{1}{r} + r = \frac{21}{a} - 1$$

$$\frac{1}{r} + r = \frac{21}{6} - 1$$

$$\frac{1 + r^2}{r} = \frac{7}{2} - 1$$

$$\frac{1 + r^2}{r} = \frac{5}{2}$$

$$2 + 2r^2 = 5r$$

$$2r^2 - 5r + 2 = 0$$

$$r = 2 \; , \; r = \frac{1}{2}$$

Hence

$a = 6$, $r = 2$	$a = 6$, $r = \frac{1}{2}$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{3 \; , \; 6 \; , \; 12}$	$\boxed{12 \; , \; 6 \; , \; 3}$

07. sum is $^{13}/_3$ & sum of squares is $^{91}/_9$

let 3 nos. in GP be $\frac{a}{r}$, a , ar

sum = 13/3

$$\frac{a}{r} + a + ar = \frac{13}{3}$$

$$a\left(\frac{1}{r} + 1 + r\right) = \frac{13}{3}$$

$$\frac{1}{r} + 1 + r = \frac{13}{3a}$$

$$\frac{1}{r} + r = \frac{13}{3a} - 1$$

sum of squares is 91/9

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = \frac{91}{9}$$

$$a^2\left(\frac{1}{r^2} + 1 + r^2\right) = \frac{91}{9}$$

$$\frac{1}{r^2} + 1 + r^2 = \frac{91}{9a^2}$$

$$\frac{1}{r^2} + r^2 = \frac{91}{9a^2} - 1$$

NOW

$$\frac{1}{r^2} + r^2 = \left(\frac{1}{r} + r\right)^2 - 2$$

$$\frac{91}{9a^2} - 1 = \left(\frac{13}{3a} - 1\right)^2 - 2$$

$$\frac{91}{9a^2} - 1 = \frac{169}{9a^2} - \frac{26}{3a} + 1 - 2$$

$$\frac{91}{9a^2} = \frac{169}{9a^2} - \frac{26}{3a}$$

$$\frac{91}{9a^2} = \frac{169 - 78a}{9a^2}$$

$$91 = 169 - 78a$$

$$78a = 78$$

$$a = 1$$

subs a = 1 in

$$\frac{1}{r} + r = \frac{13}{3a} - 1$$

$$\frac{1}{r} + r = \frac{13}{3} - 1$$

$$\frac{1 + r^2}{r} = \frac{10}{3}$$

$$3 + 3r^2 = 10r$$

$$3r^2 - 10r + 3 = 0$$

$$r = 3 \;,\; r = 1/3$$

Hence

$a = 1$, $r = 3$	$a = 1$, $r = \frac{1}{3}$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{\frac{1}{3} , 1 , 3}$	$\boxed{3 , 1 , \frac{1}{3}}$

08. sum is 26 & sum of squares is 364

let 3 nos. in GP be $\frac{a}{r}$, a , ar

sum = 26

$$\frac{a}{r} + a + ar = 26$$

$$a\left(\frac{1}{r} + 1 + r\right) = 26$$

$$\frac{1}{r} + 1 + r = \frac{26}{a}$$

$$\frac{1}{r} + r = \frac{26}{a} - 1$$

sum of squares is 364

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = 364$$

$$a^2\left(\frac{1}{r^2} + 1 + r^2\right) = 364$$

$$\frac{1}{r^2} + 1 + r^2 = \frac{364}{a^2}$$

$$\frac{1}{r^2} + r^2 = \frac{364}{a^2} - 1$$

NOW

$$\frac{1}{r^2} + r^2 = \left(\frac{1}{r} + r\right)^2 - 2$$

$$\frac{364}{a^2} - 1 = \left(\frac{26}{a} - 1\right)^2 - 2$$

$$\frac{364}{a^2} - 1 = \frac{676}{a^2} - \frac{52}{a} + 1 - 2$$

$$\frac{364}{a^2} = \frac{676}{a^2} - \frac{52}{a}$$

$$\frac{364}{a^2} = \frac{676 - 52a}{a^2}$$

$$364 = 676 - 52a$$

$$52a = 312$$

$$a = 6$$

subs $a = 6$ in

$$\frac{1}{r} + r = \frac{26}{a} - 1$$

$$\frac{1}{r} + r = \frac{26}{6} - 1$$

$$\frac{1 + r^2}{r} = \frac{13}{3} - 1$$

$$\frac{1 + r^2}{r} = \frac{10}{3}$$

$$3 + 3r^2 = 10r$$

$$3r^2 - 10r + 3 = 0$$

$$r = 3 \ , \ r = 1/3$$

Hence

$a = 6$, $r = 3$	$a = 6$, $r = \frac{1}{3}$
$\frac{a}{r}$, a , ar	$\frac{a}{r}$, a , ar
$\boxed{2 \ , \ 6 \ , \ 18}$	$\boxed{18, \ 6 \ , \ 2}$

09.

find 5 numbers in GP such that product = 1024 and sum of second and fourth is 10

let 5 nos. in GP be $\frac{a}{r^2}$, $\frac{a}{r}$, a , ar , ar^2

Product = 1024

$$\frac{a}{r^2} . \frac{a}{r} . a . ar.ar^2 = 1024$$

$$a^5 = 1024$$

$$a = 4$$

sum of second and fourth is 10

$$\frac{a}{r} + ar = 10$$

$$a\left(\frac{1}{r} + r\right) = 10$$

$$\frac{1}{r} + r = \frac{10}{a}$$

$$\frac{1}{r} + r = \frac{10}{4}$$

$$\frac{1 + r^2}{r} = \frac{5}{2}$$

$$2 + 2r^2 = 5r$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r - 2) - 1(r - 2) = 0$$

$$(2r - 1)(r - 2) = 0$$

$$r = \frac{1}{2} \; ; \; r = 2$$

Hence

$a = 4$ & $r = 2$

nos. are : 1 , 2 , 4 , 8 , 16

$a = 4$ & $r = 1/2$

nos. are : 16 , 8 , 4 , 2 , 1

10.

product = 32 and product of fourth and fifth number is 108

let 5 nos. in GP be $\frac{a}{r^2}$, $\frac{a}{r}$, a , ar , ar^2

Product = 32

$$\frac{a}{r^2} . \frac{a}{r} . a . ar.ar^2 = 32$$

$$a^5 = 32$$

$$a = 2$$

product of fourth and fifth number is 108

$$ar , ar^2 = 108$$

$$a^2r^3 = 108$$

$$4.r^3 = 108$$

$$r^3 = 27 \qquad \therefore r = 3$$

nos are : $\frac{a}{r^2}$, $\frac{a}{r}$, a , ar , ar^2

$\frac{2}{9}$, $\frac{2}{3}$, 2 , 6 , 18

11.

find 4 nos in GP such that
product is 1 and sum of middle two nos is 10/3

let 4 nos. in GP be $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3

Product = 1

$$a^4 = 1$$

$$a = 1$$

sum of middle two numbers is 10/3

$$\frac{a}{r} + ar = \frac{10}{3}$$

$$a\left(\frac{1}{r} + r\right) = \frac{10}{3}$$

$$\frac{1}{r} + r = \frac{10}{3}$$

$$\frac{1}{r} + r = \frac{10}{3}$$

$$\frac{1 + r^2}{r} = \frac{10}{3}$$

$$3 + 3r^2 = 10r$$

$$3r^2 - 10r + 3 = 0$$

$$r = 3 \ , \ r = {}^1/_3$$

Hence

$a = 1$, $r = 3$

Numbers are $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3

$\frac{1}{27}$, $\frac{1}{3}$, 3 , 27

$a = 1$, $r = {}^1/_3$

Numbers are $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3

27 , 3 , $\frac{1}{3}$, $\frac{1}{27}$

12.

find 4 nos in GP such that
product is 729 and sum of middle two nos is 12

let 4 nos. in GP be $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3

Product = 729

$$a^4 = 729$$

$$a^2 = 27$$

$$a = 3\sqrt{3}$$

sum of middle two numbers is 12

$$\frac{a}{r} + ar = \underline{12}$$

$$a\left(\frac{1}{r} + r\right) = \underline{12}$$

$$\frac{1}{r} + r = \frac{12}{3\sqrt{3}}$$

$$\frac{1}{r} + r = \frac{4}{\sqrt{3}}$$

$$\frac{1 + r^2}{r} = \frac{4}{\sqrt{3}}$$

$$\sqrt{3} + \sqrt{3}r^2 = 4r$$

$$\sqrt{3}r^2 - 4r + \sqrt{3} = 0$$

$$\sqrt{3}r^2 - 3r - 1r + \sqrt{3} = 0$$

$$\sqrt{3}r(r - \sqrt{3}) - 1(r - \sqrt{3}) = 0$$

$$(r - \sqrt{3})(\sqrt{3}r - 1) = 0$$

$$r = \sqrt{3} \ , \ r = {}^1/_{\sqrt{3}}$$

Hence

$a = 3\sqrt{3}$, $r = \sqrt{3}$

Numbers are $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3

1 , 3 , 9 , 27

$a = 1$, $r = {}^1/_3$

Numbers are $\frac{a}{r^3}$, $\frac{a}{r}$, ar , ar^3

27 , 9 , 3 , 1

SOLUTION - QSET 3

01. in G.P. $t_3 = 36$; $t_6 = 972$. Find S_4

$t_3 = 36 \therefore ar^2 = 36$ ...... (1)

$t_6 = 972 \therefore ar^5 = 972$ ...... (2)

$$\frac{ar^5}{ar^2} = \frac{972}{36}$$

$r^3 = 27$

$r = 3$

subs in (1)

$a(3)^2 = 36$

$a.9 = 36$

$a = 4$

Now

$a = 4$; $r = 3$; $n = 4$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_4 = \frac{4(3^4 - 1)}{3 - 1}$$

$$= \frac{4(81 - 1)}{2}$$

$= 2(80)$

$= 160$

02. in G.P. $t_3 = 18$; $t_6 = 486$. Find S_5

$t_3 = 18 \therefore ar^2 = 18$ ...... (1)

$t_6 = 486 \therefore ar^5 = 486$ ...... (2)

$$\frac{ar^5}{ar^2} = \frac{486}{18}$$

$r^3 = 27$

$r = 3$

subs in (1)

$a(3)^2 = 18$

$a.9 = 18$

$a = 2$

Now

$a = 2$; $r = 3$; $n = 5$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{2(3^5 - 1)}{3 - 1}$$

$$= \frac{2(243 - 1)}{2}$$

$= 242$

03. in G.P. $t_4 = 24$; $t_9 = 768$. Find S_8

$t_4 = 24 \therefore ar^3 = 24$ ...... (1)

$t_9 = 972 \therefore ar^8 = 768$ ...... (2)

$$\frac{ar^8}{ar^3} = \frac{768}{24}$$

$r^5 = 32$

$r = 2$

subs in (1)

$a(2)^3 = 24$

$a.8 = 24$

$a = 3$

Now

$a = 3$; $r = 2$; $n = 8$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{3(2^8 - 1)}{2 - 1}$$

$= 3(256 - 1)$

$= 3(255) = 765$

04. in G.P. $S_3 = 125$; $S_6 = 152$. Find r

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_3 = 125 \quad \therefore \frac{a(r^3 - 1)}{r - 1} = 125 \quad ....(1)$$

$$S_6 = 152 \quad \therefore \frac{a(r^6 - 1)}{r - 1} = 152 \quad ....(2)$$

(2) ÷ (1)

$$\frac{\frac{a(r^6 - 1)}{r - 1}}{\frac{a(r^3 - 1)}{r - 1}} = \frac{152}{125}$$

$$\frac{r^6 - 1}{r^3 - 1} = \frac{152}{125}$$

$$\frac{(r^3 - 1)(r^3 + 1)}{r^3 - 1} = \frac{152}{125}$$

$$r^3 + 1 = \frac{152}{125}$$

$$r^3 = \frac{152}{125} - 1$$

$$r^3 = \frac{27}{125} \qquad r = \frac{3}{5}$$

05. in G.P. $S_4 = 81$; $S_8 = 97$. Find r

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_4 = 81 \quad \therefore \frac{a(r^4 - 1)}{r - 1} = 81 \; ....(1)$$

$$S_8 = 97 \quad \therefore \frac{a(r^8 - 1)}{r - 1} = 97 \; ....(2)$$

(2) ÷ (1)

$$\frac{\frac{a(r^8 - 1)}{r - 1}}{\frac{a(r^4 - 1)}{r - 1}} = \frac{97}{81}$$

$$\frac{r^8 - 1}{r^4 - 1} = \frac{97}{81}$$

$$\frac{(r^4 - 1)(r^4 + 1)}{r^4 - 1} = \frac{97}{81}$$

$$r^4 + 1 = \frac{97}{81}$$

$$r^4 = \frac{97}{81} - 1$$

$$r^4 = \frac{16}{81}$$

$$r^2 = \frac{4}{9}$$

$$r = \pm \frac{2}{3}$$

05. if 1 + 4 + 16 + 64 + ... n terms = 5461 . Find n

$a = 1$; $r = 4$; $S_n = 5461$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$5461 = \frac{1(4^n - 1)}{4 - 1}$$

$$5461 = \frac{4^n - 1}{3}$$

$$4^n - 1 = 16383$$

$$4^n = 16384$$

$$4^n = 4^7$$

$$n = 7$$

4	16384
4	4096
4	1024
4	256
4	64
4	16
4	4
	1

06. $\sqrt{3} + 3 + 3\sqrt{3} + \ldots\ldots = 39 + 13\sqrt{3}$.
Find n

$a = \sqrt{3}\,;\quad r = \sqrt{3}\,;\quad S_n = 39 + 13\sqrt{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$39 + 13\sqrt{3} = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$13(3 + \sqrt{3}) = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$13\cancel{\sqrt{3}}(\sqrt{3} + 1) = \frac{\cancel{\sqrt{3}}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$13(\sqrt{3} + 1) = \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1}$$

$$13(\sqrt{3} + 1)(\sqrt{3} - 1) = \sqrt{3}^n - 1$$

$$13(3 - 1) = \sqrt{3}^n - 1$$

$$26 = \sqrt{3}^n - 1$$

$$27 = \sqrt{3}^n$$

$$3^3 = \sqrt{3}^n$$

$$\sqrt{3}^6 = \sqrt{3}^n$$

$$n = 6$$

07. $\sqrt{3} + 3 + 3\sqrt{3} + \ldots\ldots = 120 + 40\sqrt{3}$.
Find n

$a = \sqrt{3}\,;\quad r = \sqrt{3}\,;\quad S_n = 120 + 40\sqrt{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$120 + 40\sqrt{3} = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$40(3 + \sqrt{3}) = \frac{\sqrt{3}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$40\cancel{\sqrt{3}}(\sqrt{3} + 1) = \frac{\cancel{\sqrt{3}}(\sqrt{3}^n - 1)}{\sqrt{3} - 1}$$

$$40(\sqrt{3} + 1) = \frac{\sqrt{3}^n - 1}{\sqrt{3} - 1}$$

$$40(\sqrt{3} + 1)(\sqrt{3} - 1) = \sqrt{3}^n - 1$$

$$40(3 - 1) = \sqrt{3}^n - 1$$

$$80 = \sqrt{3}^n - 1$$

$$81 = \sqrt{3}^n$$

$$3^4 = \sqrt{3}^n$$

$$\sqrt{3}^8 = \sqrt{3}^n$$

$$n = 8$$

SOLUTION - QSET 4

Q1. Find Sn

01.

$9 + 99 + 999 + \ldots\ldots\ldots$

$= (10 - 1) + (100 - 1) + (1000 - 1) \ldots\ldots.$

$= (10 + 100 + 1000 + \ldots..) - (1 + 1 + 1 + \ldots..)$

$=$ (Sn of GP : $a = 10$, $r = 10$) $-$ n

$$= \frac{a(r^n - 1)}{r - 1} - n$$

$$= \frac{10(10^n - 1)}{10 - 1} - n$$

$$= \frac{10(10^n - 1)}{9} - n$$

$$= \frac{10(10^n - 1) - 9n}{9}$$

02.

$7 + 77 + 777 + \ldots\ldots\ldots$

$= 7 \ (1 + 11 + 111 + \ldots\ldots\ldots.)$

$= \frac{7}{9} \ (9 + 99 + 999 + \ldots\ldots\ldots.)$

$$= \frac{7}{9}\left[(10 - 1) + (100 - 1) + (1000 - 1) \ldots\ldots.\right]$$

$$= \frac{7}{9}\left[(10 + 100 + 1000 + \ldots..) - (1 + 1 + 1 + \ldots..)\right]$$

$$= \frac{7}{9}\left[(\text{Sn of GP} : a = 10 , r = 10) - n\right]$$

$$= \frac{7}{9}\left[\frac{a(r^n - 1)}{r - 1} - n\right]$$

$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

$$= \frac{7}{81}\left[10(10^n - 1) - 9n\right]$$

03.

$8 + 88 + 888 + \ldots\ldots\ldots$

$= 8 \ (1 + 11 + 111 + \ldots\ldots\ldots.)$

$= \frac{8}{9} \ (9 + 99 + 999 + \ldots\ldots\ldots.)$

$$= \frac{8}{9}\left[(10 - 1) + (100 - 1) + (1000 - 1) \ldots\ldots.\right]$$

$$= \frac{8}{9}\left[(10 + 100 + 1000 + \ldots..) - (1 + 1 + 1 + \ldots..)\right]$$

$$= \frac{8}{9}\left[(\text{Sn of GP} : a = 10 , r = 10) - n\right]$$

$$= \frac{8}{9}\left[\frac{a(r^n - 1)}{r - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

$$= \frac{8}{81}\left[10(10^n - 1) - 9n\right]$$

Q2. Find Sn

01.

$0.9 + 0.99 + 0.999 + \ldots\ldots\ldots$

$= (1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots\ldots$

$= (1 + 1 + 1 + \ldots.) - (0.1 + 0.01 + 0.001 + \ldots)$

$= n -$ (Sn of GP : $a = 0.1$, $r = 0.1$)

$$= n - \frac{a(1 - r^n)}{1 - r}$$

$$= n - \frac{0.1(1 - 0.1^n)}{1 - 0.1}$$

$$= n - \frac{0.1(1 - 0.1^n)}{0.9}$$

$$= n - \frac{1(1-0.1^n)}{9}$$

$$= \frac{9n - 1 + 0.1^n}{9}$$

02.

$0.8 + 0.88 + 0.888 + \dots\dots\dots$

$= 8 \ (0.1 + 0.11 + 0.111 + \dots\dots\dots)$

$= \frac{8}{9} \ (0.9 + 0.99 + 0.999 + \dots\dots\dots)$

$= \frac{8}{9}\left[(1-0.1) + (1-0.01) + (1-0.001) + \dots\dots\right]$

$= \frac{8}{9}\left[(1+1+1+\dots.) - (0.1 + 0.01 + 0.001 + \dots)\right]$

$= \frac{8}{9}\left[n - \ (S_n \text{ of GP} : a = 0.1 \ , \ r = 0.1)\right]$

$= \frac{8}{9}\left[n - \frac{a(1-r^n)}{1-r}\right]$

$= \frac{8}{9}\left[n - \frac{0.1(1-0.1^n)}{1-0.1}\right]$

$= \frac{8}{9}\left[n - \frac{0.1(1-0.1^n)}{0.9}\right]$

$= \frac{8}{9}\left[n - \frac{1(1-0.1^n)}{9}\right]$

$= \frac{8}{81}\left[9n - 1 + 0.1^n\right]$

03.

$0.3 + 0.33 + 0.333 + \dots\dots\dots$

$= 3 \ (0.1 + 0.11 + 0.111 + \dots\dots\dots)$

$= \frac{3}{9} \ (0.9 + 0.99 + 0.999 + \dots\dots\dots)$

$= \frac{1}{3}\left[(1-0.1) + (1-0.01) + (1-0.001) + \dots\dots\right]$

$= \frac{1}{3}\left[(1+1+1+\dots.) - (0.1 + 0.01 + 0.001 + \dots)\right]$

$= \frac{1}{3}\left[n - \ (S_n \text{ of GP} : a = 0.1 \ , \ r = 0.1)\right]$

$= \frac{1}{3}\left[n - \frac{a(1-r^n)}{1-r}\right]$

$= \frac{1}{3}\left[n - \frac{0.1(1-0.1^n)}{1-0.1}\right]$

$= \frac{1}{3}\left[n - \frac{0.1(1-0.1^n)}{0.9}\right]$

$= \frac{1}{3}\left[n - \frac{1(1-0.1^n)}{9}\right]$

$= \frac{1}{27}\left[9n - 1 + 0.1^n\right]$

Q3. Find tn

01.

$0.3 \ , \ 0.33 \ , \ 0.333 \ , \ \dots\dots\dots$

$t_1 = 0.3$

$t_2 = 0.33$
$\quad = 0.3 + 0.03$

$t_3 = 0.333$
$\quad = 0.3 + 0.03 + 0.003$

$t_4 = 0.3333$
$\quad = 0.3 + 0.03 + 0.003 + 0.0003$

Hence

$t_n = 0.3 + 0.03 + 0.003 + \dots.. \text{ n terms}$

$\quad = S_n \text{ of GP} : a = 0.3 \ , \ r = 0.1$

$\quad = \frac{a(1-r^n)}{1-r}$

$\quad = \frac{0.3\ (1-0.1^n)}{1-0.1}$

$\quad = \frac{0.3(1-0.1^n)}{0.9}$

$\quad = \frac{3}{9}(1-0.1^n) \qquad = \frac{1}{3}(1-0.1^n)$

02.

0.4 , 0.44 , 0.444 , ...

$t_1 = 0.4$

$t_2 = 0.44$

$= 0.4 + 0.04$

$t_3 = 0.444$

$= 0.4 + 0.04 + 0.004$

$t_4 = 0.4444$

$= 0.4 + 0.04 + 0.004 + 0.0004$

Hence

$t_n = 0.4 + 0.04 + 0.004 + \ldots$ n terms

$=$ Sn of GP : $a = 0.4$, $r = 0.1$

$$= \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{0.4\ (1 - 0.1^n)}{1 - 0.1}$$

$$= \frac{0.4(1 - 0.1^n)}{0.9}$$

$$= \frac{4}{9}(1 - 0.1^n) \qquad = \frac{4}{9}(1 - 0.1^n)$$

03.

0.7 , 0.77 , 0.777 , ...

$t_1 = 0.7$

$t_2 = 0.77$

$= 0.7 + 0.07$

$t_3 = 0.777$

$= 0.7 + 0.07 + 0.007$

$t_4 = 0.7777$

$= 0.7 + 0.07 + 0.007 + 0.0007$

Hence

$t_n = 0.4 + 0.04 + 0.004 + \ldots$ n terms

$=$ Sn of GP : $a = 0.4$, $r = 0.1$

$$= \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{0.7\ (1 - 0.1^n)}{1 - 0.1}$$

$$= \frac{0.7(1 - 0.1^n)}{0.9}$$

$$= \frac{7}{9}(1 - 0.1^n) \qquad = \frac{7}{9}(1 - 0.1^n)$$

SOLUTION - QSET 5

Q1.

01. if for a sequence ; $t_n = \dfrac{2^{n-2}}{5^{n-3}}$;

show that sequence is a G.P. Find its first term and the common ratio

$$t_n = \frac{2^{n-2}}{5^{n-3}}$$

$$t_{n-1} = \frac{2^{n-1-2}}{5^{n-1-3}}$$

$$= \frac{2^{n-3}}{5^{n-4}}$$

$$\frac{t_n}{t_{n-1}} = \frac{\frac{2^{n-2}}{5^{n-3}}}{\frac{2^{n-3}}{5^{n-4}}}$$

$$\frac{t_n}{t_{n-1}} = \frac{2^{n-2}}{5^{n-3}} \times \frac{5^{n-4}}{2^{n-3}}$$

$$= \frac{2^{n-2}}{2^{n-3}} \times \frac{5^{n-4}}{5^{n-3}}$$

$$= \frac{2^{n-2-n+3}}{5^{n-3-n+4}}$$

$$= \frac{2}{5}$$

= CONSTANT . Hence G.P.

$$r = \frac{2}{5}$$

$$a = t_1 = \frac{2^{1-2}}{5^{1-3}}$$

$$= \frac{2^{-1}}{5^{-2}}$$

$$= \frac{5^2}{2}$$

$$= \frac{25}{2}$$

02. if for a sequence ; $t_n = \dfrac{4^{n-3}}{5^{n-2}}$;

show that sequence is a G.P. Find its first term and the common ratio

$$t_n = \frac{4^{n-3}}{5^{n-2}}$$

$$t_{n-1} = \frac{4^{n-1-3}}{5^{n-1-2}}$$

$$= \frac{4^{n-4}}{5^{n-3}}$$

$$\frac{t_n}{t_{n-1}} = \frac{\frac{4^{n-3}}{5^{n-2}}}{\frac{4^{n-4}}{5^{n-3}}}$$

$$\frac{t_n}{t_{n-1}} = \frac{4^{n-3}}{5^{n-2}} \times \frac{5^{n-3}}{4^{n-4}}$$

$$= \frac{4^{n-3}}{4^{n-4}} \times \frac{5^{n-3}}{5^{n-2}}$$

$$= \frac{4^{n-3-n+4}}{5^{n-2-n+3}}$$

$$= \frac{4}{5}$$

= CONSTANT . Hence G.P.

$$r = \frac{4}{5}$$

$$a = t_1 = \frac{4^{1-3}}{5^{1-2}}$$

$$= \frac{4^{-2}}{5^{-1}}$$

$$= \frac{5^1}{4^2}$$

$$= \frac{5}{16}$$

03. if for a sequence ; $t_n = \dfrac{7^{n+3}}{5^{n+2}}$;

show that sequence is a G.P. Find its first term and the common ratio

$$t_n = \frac{7^{n+3}}{5^{n+2}}$$

$$t_{n-1} = \frac{7^{n-1+3}}{5^{n-1+2}}$$

$$= \frac{7^{n+2}}{5^{n+1}}$$

$$\frac{t_n}{t_{n-1}} = \frac{\frac{7^{n+3}}{5^{n+2}}}{\frac{7^{n+2}}{5^{n+1}}}$$

$$\frac{t_n}{t_{n-1}} = \frac{7^{n+3}}{5^{n+2}} \times \frac{5^{n+1}}{7^{n+2}}$$

$$= \frac{7^{n+3}}{7^{n+2}} \times \frac{5^{n+1}}{5^{n+2}}$$

$$= \frac{7^{n+3-n-2}}{7^{n+2-n-1}}$$

$$= \frac{7}{5}$$

= CONSTANT . Hence G.P.

$$r = \frac{7}{5}$$

$$a = t_1 = \frac{7^{1+3}}{5^{1+2}}$$

$$= \frac{7^4}{5^3}$$

$$= \frac{2401}{125}$$

$$= \frac{5}{16}$$

Q2.

01. for a sequence ; if $S_n = 5(2^n - 1)$; find t_n and show that the sequence is a G.P.

$$S_n = 5(2^n - 1)$$

$$S_{n-1} = 5(2^{n-1} - 1)$$

$$t_n = S_n - S_{n-1}$$

$$= 5(2^n - 1) - 5(2^{n-1} - 1)$$

$$= 5\left[(2^n - 1) - (2^{n-1} - 1)\right]$$

$$= 5 \quad (2^n - 1 - 2^{n-1} + 1)$$

$$= 5 \quad (2^n - 2^{n-1})$$

$$= 5.2^n \ (1 - 2^{-1})$$

$$= 5.2^n \left(1 - \frac{1}{2}\right)$$

$$t_n = \frac{5}{2} . 2^n$$

$$t_{n-1} = \frac{5}{2} . 2^{n-1}$$

$$\frac{t_n}{t_{n-1}} = \frac{2^n}{2^{n-1}}$$

$$= 2^{n-n+1}$$

$$= 2$$

= CONSTANT

Hence G.P.

02. for a sequence ; if $S_n = 3(2^n - 1)$; find t_n and show that the sequence is a G.P.

$S_n = 3(2^n - 1)$

$S_{n-1} = 3(2^{n-1} - 1)$

$t_n = S_n - S_{n-1}$

$= 3(2^n - 1) - 3(2^{n-1} - 1)$

$= 3\left[(2^n - 1) - (2^{n-1} - 1)\right]$

$= 3 \ (2^n - 1 - 2^{n-1} + 1)$

$= 3 \ (2^n - 2^{n-1})$

$= 3.2^n \ (1 - 2^{-1})$

$= 3.2^n \left(1 - \frac{1}{2}\right)$

$t_n = \frac{3}{2} . 2^n$

$t_{n-1} = \frac{3}{2} . 2^{n-1}$

$\frac{t_n}{t_{n-1}} = \frac{2^n}{2^{n-1}}$

$= 2^{n-n+1}$

$= 2$

$=$ CONSTANT

Hence G.P.

03. for a sequence ; if $S_n = \frac{4^n - 3^n}{3^n}$;

find t_n and show that the sequence is a G.P

$S_n = \frac{4^n - 3^n}{3^n}$

$S_n = \left(\frac{4}{3}\right)^n - 1$

$S_{n-1} = \left(\frac{4}{3}\right)^{n-1} - 1$

$t_n = S_n - S_{n-1}$

$= \left\{\left(\frac{4}{3}\right)^n - 1\right\} - \left\{\left(\frac{4}{3}\right)^{n-1} - 1\right\}$

$= \left(\frac{4}{3}\right)^n - 1 - \left(\frac{4}{3}\right)^{n-1} + 1$

$= \left(\frac{4}{3}\right)^n - \left(\frac{4}{3}\right)^{n-1}$

$= \left(\frac{4}{3}\right)^n \left\{1 - \left(\frac{4}{3}\right)^{-1}\right\}$

$= \left(\frac{4}{3}\right)^n \left(1 - \frac{3}{4}\right)$

$t_n = \frac{1}{4}\left(\frac{4}{3}\right)^n$

$t_{n-1} = \frac{1}{4}\left(\frac{4}{3}\right)^{n-1}$

$\frac{t_n}{t_{n-1}} = \frac{\frac{1}{4}\left(\frac{4}{3}\right)^n}{\frac{1}{4}\left(\frac{4}{3}\right)^{n-1}} = \left(\frac{4}{3}\right)^{n-n+1}$

$= \left(\frac{4}{3}\right) =$ CONSTANT . $\therefore$ GP .

INFINITE GEOMETRIC PROGRESSION

If $|r| < 1$; $S_\infty = \dfrac{a}{1-r}$

Q1.

01. if the first term of G.P. is 2 and sum to infinity is 6 , find the common ratio

02. the sum of an infinite G.P. is 80/9 and its common ratio is $-4/5$, find the first term

03. second term of an infinite geometric series is 3/2 and its sum is 8 . Find the series

04. in a GP sum of infinite terms is 15 and sum of squares of these infinite terms is 45 . Find the series

Q2. Express the following recurring decimals as rational numbers (in the form of p/q)

01. $0.\overline{23}$

02. $0.1\overline{42}$

03. $0.2\overline{34}$

04. $2.3\overline{56}$

Q1.

01. if the first term of G.P. is 2 and sum to infinity is 6 , find the common ratio

$a = 2$; $S_\infty = 6$

$$S_\infty = \frac{a}{1-r}$$

$$6 = \frac{2}{1-r}$$

$$1 - r = \frac{1}{3}$$

$$r = 1 - \frac{1}{3} = \frac{2}{3}$$

02. the sum of an infinite G.P. is 80/9 and its common ratio is $-4/5$, find the first term

$r = -\dfrac{4}{5}$; $S_\infty = \dfrac{80}{9}$

$$S_\infty = \frac{a}{1-r}$$

$$\frac{80}{9} = \frac{a}{1 + \frac{4}{5}}$$

$$\frac{80}{9} = \frac{a}{\frac{9}{5}}$$

$$a = \frac{80}{9} \times \frac{9}{5}$$

$$a = 16$$

03. second term of an infinite geometric series is 3/2 and its sum is 8 . Find the series

$ar = \frac{3}{2}$ ....(1) ; $S\infty = 8$

$S\infty = \frac{a}{1-r}$

$8 = \frac{a}{1-r}$

$8 - 8r = a$

$8 - 8r = \frac{3}{2r}$ ....From (1)

$16r - 16r^2 = 3$

$16r^2 - 16r + 3 = 0$

$16r^2 - 12r - 4r + 3 = 0$

$4r(4r - 3) - 1(4r - 3) = 0$

$(4r - 1)(4r - 3) = 0$

$r = \frac{1}{4}$; $r = \frac{3}{4}$

NOW: subs $r = \frac{1}{4}$ in (1)

$a \cdot \frac{1}{4} = \frac{3}{2}$

$a = 6$

series : a , ar , ar^2, ...

6 , $\frac{3}{2}$, $\frac{3}{8}$, ...

NOW : subs $r = \frac{3}{4}$ in (1)

$a \cdot \frac{3}{4} = \frac{3}{2}$

$a = 2$

series : a , ar , ar^2, ...

2 , $\frac{3}{2}$, $\frac{3}{16}$, ...

04. in a GP sum of infinite terms is 15 and sum of squares of these infinite terms is 45 . Find the series

$a + ar + ar^2 + \ldots\ldots \infty = 15$

$\frac{a}{1-r} = 15$ ....(1)

$a^2 + a^2r^2 + \ldots\ldots \infty = 45$

$\frac{a^2}{1-r^2}. = 45$ ....(2)

$\frac{15(1-r)^2}{1-r^2.} = 45$

$\frac{225(1-r)^2}{(1-r)(1+r)} = 45$

$\frac{225(1-r)}{1+r} = 45$

$\frac{1+r}{1-r} = \frac{1}{5}$

$5 + 5r = 1 - r$

$6r = 4$ $\therefore r = 2/3$

subs in (1) $\frac{a}{1-\frac{2}{3}} = 15$

$\frac{a}{\frac{1}{3}} = 15$

$3a = 15$

$\therefore a = 5$

$\therefore$ series : $a = 5$; $r = \frac{2}{3}$

a , ar , ar^2 , ...

5 , $\frac{10}{3}$, $\frac{20}{9}$

Q2. Express the following recurring decimals as rational numbers (in the form of p/q)

01. $0.\overline{23}$

$= 0.232323.....$

$= 0.23 + 0.0023 + 0.000023 + ... \infty$

$= \frac{23}{10^2} + \frac{23}{10^4} + \frac{23}{10^6} + ... \infty$

$=$ sum of infinite GP : $a = \frac{23}{10^2}$, $r = \frac{1}{10^2}$

$= \frac{a}{1 - r}$

$= \frac{\frac{23}{10^2}}{1 - \frac{1}{10^2}}$

$= \frac{\frac{23}{100}}{1 - \frac{1}{100}}$

$= \frac{\frac{23}{100}}{\frac{99}{100}} \quad = \frac{23}{99}$

02. $0.1\dot{4}\dot{2}$

$= 0.1424242.....$

$= 0.1 + 0.042 + 0.00042 + ... \infty$

$= \frac{1}{10} + \frac{42}{10^3} + \frac{42}{10^5} + ... \infty$

$= \frac{1}{10} +$ $S\infty$: $a = \frac{42}{10^3}$, $r = \frac{1}{10^2}$

$= \frac{1}{10} + \frac{a}{1 - r}$

$= \frac{1}{10} + \frac{\frac{42}{10^3}}{1 - \frac{1}{10^2}}$

$= \frac{1}{10} + \frac{\frac{42}{1000}}{1 - \frac{1}{100}}$

$= \frac{1}{10} + \frac{\frac{42}{1000}}{\frac{99}{100}}$

$= \frac{1}{10} + \frac{42}{990}$

$= \frac{99 + 42}{990} \quad = \frac{141}{990}$

03. $0.2\overline{34}$

$= 0.2343434.....$

$= 0.2 + 0.034 + 0.00034 + ... \infty$

$= \frac{2}{10} + \frac{34}{10^3} + \frac{34}{10^5} + ... \infty$

$= \frac{2}{10} +$ $S\infty$: $a = \frac{34}{10^3}$, $r = \frac{1}{10^2}$

$= \frac{2}{10} + \frac{a}{1 - r}$

$= \frac{2}{10} + \frac{\frac{34}{10^3}}{1 - \frac{1}{10^2}}$

$= \frac{2}{10} + \frac{\frac{34}{1000}}{1 - \frac{1}{100}}$

$= \frac{2}{10} + \frac{\frac{34}{1000}}{\frac{99}{100}}$

$= \frac{2}{10} + \frac{34}{990}$

$= \frac{198 + 34}{990}$

$= \frac{232}{990} \quad = \frac{116}{495}$

04. $2.3\overline{56}$

$= 2.3565656.....$

$= 2.3 + 0.056 + 0.00056 + ... \infty$

$= \frac{23}{10} + \frac{56}{10^3} + \frac{56}{10^5} + ... \infty$

$= \frac{23}{10} + \quad S_\infty : a = \frac{56}{10^3} \quad , r = \frac{1}{10^2}$

$= \frac{23}{10} + \frac{a}{1 - r}$

$$= \frac{23}{10} + \frac{\frac{56}{10^3}}{1 - \frac{1}{10^2}}$$

$$= \frac{23}{10} + \frac{\frac{56}{1000}}{1 - \frac{1}{100}}$$

$$= \frac{23}{10} + \frac{\frac{56}{1000}}{\frac{99}{100}}$$

$= \frac{23}{10} + \frac{56}{990}$

$= \frac{23(99) + 56}{990}$

$= \frac{2277 + 56}{990}$

$= \frac{2333}{990}$

HARMONIC PROGRESSION

If 1 , 4 , 7 , 10 are in A.P.

then their reciprocals

$\frac{1}{1}$, $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{10}$, ... are in H.P.

NOTE : In general there is no formula for H.P. . Questions on H.P. are generally solved by inverting terms and making use of corresponding A.P.

MEANS

ARITHMETIC MEAN (A.M.)

If three numbers a , A , b are in A.P. then 'A' is called the arithmetic mean

$$A = \frac{a + b}{2}$$

GEOMETRIC MEAN (G.M.)

If three numbers a , G , b are in G.P. then 'G' is called the Geometric mean

$$G^2 = ab$$

HARMONIC MEAN (H.M.)

If three numbers a , H , b are in H.P. then 'H' is called the harmonic mean

$$G^2 = AH$$

Q–SET

01. If A.M. of two numbers exceeds their G.M. by 10 and their H.M. by 16 , find the numbers

02. If A.M. of two numbers exceeds their G.M. by 15 and their H.M. by 27 , find the numbers

03. If A.M. of two numbers exceeds their G.M. by 30 and their H.M. by 48 , find the numbers

04. Find two numbers whose sum is 100 and the ratio of whose AM to GM is 5 : 4

01. If A.M. of two numbers exceeds their G.M. by 10 and their H.M. by 16 , find the numbers

$A - G = 10 \quad \therefore G = A - 10 \ldots\ldots (1)$

$A - H = 16 \quad \therefore H = A - 16 \ldots. (2)$

$G^2 = AH$

$(A - 10)^2 = A(A - 16)$

$A^2 - 20A + 100 = A^2 - 16A$

$100 = 4A$

$25 = A$

SUBS IN (1)

$G = 25 - 10 = 15$

NOW ;

$A = 25$	$G = 15$
$\frac{a+b}{2} = 25$	$G^2 = 225$
$a + b = 50 \ldots\ldots (3)$	$ab = 225 \ldots\ldots\ldots (4)$

Solving

$a(50 - a) = 225$

$50a - a^2 = 225$

$a^2 - 50a + 225 = 0$

$a^2 - 45a - 5a + 225 = 0$

$a(a - 45) - 5(a - 45) = 0$

$(a - 5)(a - 45) = 0$

$a = 5 \quad ; \quad a = 45$

subs in (3)

$b = 45 \quad ; \quad b = 5$ ans : 5, 45

02. If A.M. of two numbers exceeds their G.M. by 15 and their H.M. by 27 , find the numbers

$A - G = 15 \quad \therefore G = A - 15 \ldots\ldots (1)$

$A - H = 27 \quad \therefore H = A - 27 \ldots. (2)$

$G^2 = AH$

$(A - 15)^2 = A(A - 27)$

$A^2 - 30A + 225 = A^2 - 27A$

$225 = 3A$

$75 = A$

SUBS IN (1)

$G = 75 - 15 = 60$

NOW ;

$A = 75$	$G = 60$
$\frac{a+b}{2} = 75$	$G^2 = 3600$
$a + b = 150 \ldots\ldots (3)$	$ab = 3600 \ldots\ldots\ldots (4)$

Solving

$a(150 - a) = 3600$

$150a - a^2 = 3600$

$a^2 - 150a + 3600 = 0$

$a^2 - 120a - 30a + 3600 = 0$

$a(a - 120) - 30(a - 120) = 0$

$(a - 30)(a - 120) = 0$

$a = 30 \quad ; \quad a = 120$

subs in (3)

$b = 120 \quad ; \quad b = 30$ ans : 30, 120

03. If A.M. of two numbers exceeds their G.M. by 30 and their H.M. by 48, find the numbers

$A - G = 30 \quad \therefore \; G = A - 30 \; ..... \; (1)$

$A - H = 48 \quad \therefore \; H = A - 48 \; .... \; (2)$

$G^2 = AH$

$(A - 30)^2 = A(A - 48)$

$A^2 - 60A + 900 = A^2 - 48A$

$900 = 12A$

$A = 75$

SUBS IN (1)

$G = 75 - 30 = 45$

NOW ;

$A = 75$	$G = 45$
$\frac{a+b}{2} = 75$	$G^2 = 2025$
$a + b = 150$ ..... (3)	$ab = 2025$ .... (4)

Solving

$a(150 - a) = 2025$

$150a - a^2 = 2025$

$a^2 - 150a + 2025 = 0$

$$a = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{150 \pm \sqrt{22500 - 4(2025)}}{2(1)}$$

$$= \frac{150 \pm 14400}{2}$$

$$= \frac{150 \pm 120}{2}$$

$$= \frac{270}{2} , \frac{30}{2}$$

$= 135 , 15$

$a = 15 \quad ; \quad a = 135$

subs in (3)

$b = 135 \quad ; \quad b = 15$ ans : 15, 135

04. Find two numbers whose sum is 100 and the ratio of whose AM to GM is 5 : 4
Let the numbers be a & b

$a + b = 100$ ...... (1)

$$\frac{A.M.}{G.M} = \frac{5}{4}$$

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = 5/4$$

$\frac{50}{\sqrt{ab}} = \frac{5}{4}$ ...... from 1

$\sqrt{ab} = 40$

$ab = 1600$

$a(100 - a) = 1600$

$100a - a^2 = 1600$

$a^2 - 100a - 1600 = 0$

$(a - 80)(a - 20) = 0$

$a = 80 ; \quad a = 20$

$b = 20 ; \quad b = 80$

two numbers are : 20, 80

SERIES

$$1 + 2 + 3 + 4 + \dots\dots + n = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Q SET - 1

01. $1.3 + 2.4 + 3.5 + \dots$ ans : $\dfrac{n(n+1)(2n+7)}{6}$

02. $1.3 + 5.7 + 9.11 + \dots$ ans : $n\left(\dfrac{16n^2 - 7}{3}\right)$

03. $3.7 + 5.10 + 7.13 + \dots$ ans : $\dfrac{n(4n^2 + 17n + 21)}{2}$

04. $1.2.3 + 2.3.4 + 3.4.5 + \dots$ ans : $\dfrac{n(n+1)(n+2)(n+3)}{4}$

05. $1.3.5 + 2.4.6 + 3.5.7 + \dots$ ans : $\dfrac{n(n+1)(n+4)(n+5)}{4}$

06. $\Sigma\ (2r - 1)(2r + 1)$ ans : $n(4n^2 + 6n - 1)/3$

07. $\Sigma\ (6r^2 - 2r + 6)$ ans : $2n(n^2 + n + 3)$

08. $\Sigma\ (6r^2 + 4r - 3)$ ans : $n^2(2n + 5)$

09. $2^2 + 4^2 + 6^2 + \dots$ ans : $\dfrac{2n(n+1)(2n+1)}{3}$

10. $1^2 + 3^2 + 5^2 + 7^2 + \dots$ ans : $\dfrac{n(4n^2 - 1)}{3}$

Q SET - 2

01. $\frac{1}{2} + \frac{1^2 + 2^2}{3} + \frac{1^2 + 2^2 + 3^2}{4} + \ldots$ ans : $\frac{n(n + 1)(4n + 5)}{36}$

02. $\frac{1^2}{1} + \frac{1^2 + 2^2}{1 + 2} + \frac{1^2 + 2^2 + 3^2}{1 + 2 + 3} + \ldots$ ans : $\frac{n(n + 2)}{3}$

03. $\Sigma \frac{1^2 + 2^2 + 3^2 + \ldots + r^2}{r + 1}$ ans : $\frac{n(n + 1)(4n + 5)}{36}$

04. $\Sigma \frac{1^3 + 2^3 + 3^3 + \ldots + r^3}{r(r + 1)}$ ans : $\frac{n(n + 1)(n + 2)}{12}$

05. $\Sigma \frac{1^3 + 2^3 + 3^3 + \ldots + r^3}{r + 1}$ ans : $\frac{n(n+1)(n+2)(3n+1)}{48}$

Q SET - 3

01. $(40^2 - 39^2) + (38^2 - 37^2) + (36^2 - 35^2) + \ldots + (2^2 - 1^2)$

ans : 820

02. $(50^2 - 49^2) + (48^2 - 47^2) + (46^2 - 45^2) + \ldots + (2^2 - 1^2)$

ans : 1275

03. $(60^2 - 59^2) + (58^2 - 57^2) + (56^2 - 55^2) + \ldots + (2^2 - 1^2)$

ans : 1830

04. $31^2 + 32^2 + 33^2 + \ldots + 50^2$ ans : 33470

Q SET - 4

01. $\frac{1 + 3 + 5 + 7 + \ldots \text{ n terms}}{2 + 4 + 6 + 8 + \ldots \text{ n terms}} = 0.95$ **ans : n = 19**

02. $\frac{1 + 2 + 3 + 4 + \ldots \text{ n terms}}{1.2 + 2.3 + 3.4 + \ldots \text{ n terms}} = 0.03$ **ans : n = 48**

03. $\frac{1^3 + 2^3 + 3^3 + \ldots \text{ n terms}}{1.3 + 2.5 + 3.7 + \ldots \text{ n terms}} = \frac{9}{5}$ **ans : n = 5**

01. 1.3 + 2.4 + 3.5 + ...

$t_n = (n)(n+2)$

1 , 2 , 3 , ...	3 , 4 , 5 ......
$a + (n-1)d$	$a + (n-1)d$
$1 + (n-1)(1)$	$3 + (n-1)1$
$1 + n - 1$	$3 + n - 1$
n	$n + 2$

$$= \sum_{r=1}^{n} r(r+2)$$

$$= \sum (r^2 + 2r)$$

$$= \sum r^2 + 2\sum r$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{\cancel{2}\, n(n+1)}{\cancel{2}}$$

$$= n(n+1)\left(\frac{2n+1}{6} + 1\right)$$

$$= n(n+1)\left(\frac{2n+1+6}{6}\right)$$

$$= \frac{n(n+1)(2n+7)}{6}$$

02. 1.3 + 5.7 + 9.11 + ...

$t_n = (4n-3)(4n-1)$

1 , 5 , 9 , ...	3 , 7 , 11 ......
$a + (n-1)d$	$a + (n-1)d$
$1 + (n-1)(4)$	$3 + (n-1)4$
$1 + 4n - 4$	$3 + 4n - 4$
$4n - 3$	$4n - 1$

$$= \sum_{r=1}^{n} (4r-3)(4r-1)$$

$$= \sum (16r^2 - 12r - 4r + 3)$$

$$= \sum (16r^2 - 16r + 3)$$

$$= \sum 16r^2 - 16\sum r + 3\sum 1$$

$$= \frac{16n(n+1)(2n+1)}{6} - \frac{16n(n+1)}{2} + 3n$$

$$= \frac{8n(n+1)(2n+1)}{3} - 8n(n+1) + 3n$$

$$= n\left(\frac{8(n+1)(2n+1)}{3} - 8(n+1) + 3\right)$$

$$= n\left(\frac{8(2n^2+3n+1)}{3} - 8n - 8 + 3\right)$$

$$= n\left(\frac{8(2n^2+3n+1)}{3} - 8n - 5\right)$$

$$= n\left(\frac{16n^2 + 24n + 8 - 24n - 15}{3}\right)$$

$$= n\left(\frac{16n^2 - 7}{3}\right)$$

03. 3.7 + 5.10 + 7.13 + ...

$t_n = (2n-1)(3n+4)$

3 , 5 , 7 , ...	7 , 10 , 13 ......
$a + (n-1)d$	$a + (n-1)d$
$3 + (n-1)(2)$	$7 + (n-1)3$
$3 + 2n - 2$	$7 + 3n - 3$
$2n + 1$	$3n + 4$

$$= \sum_{r=1}^{n} (2r + 1)(3r + 4)$$

$$= \sum (6r^2 + 8r + 3r + 4)$$

$$= \sum (6r^2 + 11r + 4)$$

$$= \sum 6r^2 + 11 \sum r + 4 \sum 1$$

$$= \frac{6n(n + 1)(2n + 1)}{6} + 11\frac{n(n + 1)}{2} + 4n$$

$$= n(n + 1)(2n + 1) + \frac{11n(n + 1)}{2} + 4n$$

$$= n\left((n + 1)(2n + 1) + \frac{11(n+1)}{2} + 4 \right)$$

$$= n\left(2n^2 + 3n + 1 + \frac{11n + 11}{2} + 4 \right)$$

$$= n\left(\frac{4n^2 + 6n + 2 + 11n + 11 + 8}{2} \right)$$

$$= n\left(\frac{4n^2 + 17n + 21}{2} \right)$$

04. 1.2.3 + 2.3.4 + 3.4.5 + ……….

$t_n = (n)(n + 1)(n + 2)$

1 , 2 , 3 , ….	2 , 3 , 4 , …..	3 , 4 , 5 ……
$a + (n - 1)d$	$a + (n - 1)d$	$a + (n - 1)d$
$1 + (n - 1)(1)$	$2 + (n - 1)1$	$3 + (n - 1)1$
$1 + n - 1$	$2 + n - 1$	$3 + n - 1$
n	$n + 1$	$n + 2$

$$= \sum_{r=1}^{n} (r)(r + 1)(r + 2)$$

$$= \sum (r)(r^2 + 3r + 2)$$

$$= \sum (r^3 + 3r^2 + 2r)$$

$$= \sum r^3 + 3 \sum r^2 + 2 \sum r$$

$$= \frac{n^2(n + 1)^2}{4} + \frac{3n(n + 1)(2n + 1)}{6} + \frac{2n(n + 1)}{2}$$

$$= \frac{n^2(n + 1)^2}{4} + \frac{n(n + 1)(2n + 1)}{2} + n(n + 1)$$

$$= n(n + 1)\left(\frac{n(n+1)}{4} + \frac{2n + 1}{2} + 1 \right)$$

$$= n(n + 1)\left(\frac{n^2 + n}{4} + \frac{2n + 1}{2} + 1 \right)$$

$$= n(n + 1)\left(\frac{n^2 + n + 4n + 2 + 4}{4} \right)$$

$$= \frac{n(n + 1)(n^2 + 5n + 6)}{4}$$

$$= \frac{n(n + 1)(n + 2)(n + 3)}{4}$$

05. 1.3.5 + 2.4.6 + 3.5.7 + ………

$t_n = (n)(n + 1)(n + 2)$

1 , 2 , 3 , ….	3 , 4 , 5 , …..	5 , 6 , 7 ……
$a + (n - 1)d$	$a + (n - 1)d$	$a + (n - 1)d$
$1 + (n - 1)(1)$	$3 + (n - 1)1$	$5 + (n - 1)1$
$1 + n - 1$	$3 + n - 1$	$5 + n - 1$
n	$n + 2$	$n + 4$

$$= \sum_{r=1}^{n} (r)(r + 2)(r + 4)$$

$$= \sum (r)(r^2 + 6r + 8)$$

$= \sum (r^3 + 6r^2 + 8r)$

$= \sum r^3 + 6 \sum r^2 + 8 \sum r$

$= \dfrac{n^2(n+1)^2}{4} + \dfrac{6n(n+1)(2n+1)}{6} + \dfrac{8n(n+1)}{2}$

$= \dfrac{n^2(n+1)^2}{4} + n(n+1)(2n+1) + 4n(n+1)$

$= n(n+1) \left(\dfrac{n(n+1)}{4} + 2n+1 + 4\right)$

$= n(n+1) \left(\dfrac{n^2+n}{4} + 2n+5\right)$

$= n(n+1) \left(\dfrac{n^2+n+8n+20}{4}\right)$

$= \dfrac{n(n+1)(n^2+9n+20)}{4}$

$= \dfrac{n(n+1)(n+4)(n+5)}{4}$

06. $\sum_{r=1}^{n} (2r-1)(2r+1)$

$= \sum (4r^2 - 1)$

$= 4 \sum r^2 - \sum 1$

$= 4 \dfrac{n(n+1)(2n+1)}{6} - n$

$= \dfrac{2n(n+1)(2n+1)}{3} - n$

$= n\left(\dfrac{2(n+1)(2n+1)}{3} - 1\right)$

$= n\left(\dfrac{2(2n^2+n+2n+1)}{3} - 1\right)$

$= n\left(\dfrac{2(2n^2+3n+1)}{3} - 1\right)$

$= n\left(\dfrac{4n^2+6n+2}{3} - 1\right)$

$= n\left(\dfrac{4n^2+6n+2-3}{3}\right)$

$= n\left(\dfrac{4n^2+6n-1}{3}\right)$

07. $\sum (6r^2 - 2r + 6)$

$= \sum 6r^2 - 2\sum r + 6\sum 1$

$= \dfrac{6n(n+1)(2n+1)}{6} - \dfrac{2n(n+1)}{2} + 6n$

$= n(n+1)(2n+1) - n(n+1) + 6n$

$= n\left[(n+1)(2n+1) - (n+1) + 6\right]$

$= n\left[2n^2+3n+1 - n - 1 - 6\right]$

$= n\left[2n^2+2n-6\right]$

$= 2n(n^2+n-6)$

08. $\sum (6r^2 + 4r - 3)$

$= \sum 6r^2 + 4\sum r - 3\sum 1$

$= \dfrac{6n(n+1)(2n+1)}{6} + \dfrac{4n(n+1)}{2} - 3n$

$= n(n+1)(2n+1) + 2n(n+1) - 3n$

$= n\left[(n+1)(2n+1) + 2(n+1) - 3\right]$

$= n\left[2n^2+3n+1+2n+2-3\right]$

$= n\left[2n^2+5n\right]$

09. $2^2 + 4^2 + 6^2 + \dots\dots\dots$

$tn = (2n)^2$

$2, 4, 6, \dots\dots$ $\quad a + (n-1)d$

$2 + (n-1)\,2$

$2 + 2n - 2$

$2n$

$$= \sum_{r=1}^{n} (2r)^2$$

$$= \sum 4r^2$$

$$= 4\sum r^2$$

$$= \frac{4n(n+1)(2n+1)}{6}$$

$$= \frac{2n(n+1)(2n+1)}{3}$$

10. $1^2 + 3^2 + 5^2 + \dots\dots\dots$

$tn = (2n-1)^2$

$1, 3, 5, \dots\dots$ $\quad a + (n-1)d$

$1 + (n-1)\,2$

$1 + 2n - 2$

$2n - 1$

$$= \sum_{r=1}^{n} (2r-1)^2$$

$$= \sum \left(4r^2 - 4r + 1\right)$$

$$= \sum 4r^2 - 4\sum r + \sum 1$$

$$= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$$

$$= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n$$

$$= n\left(\frac{2(n+1)(2n+1)}{3} - 2(n+1) + 1\right)$$

$$= n\left(\frac{2(2n^2+3n+1)}{3} - 2n - 2 + 1\right)$$

$$= n\left(\frac{4n^2 + 6n + 2 - 6n - 1}{3}\right)$$

$$= n\left(\frac{4n^2 - 1}{3}\right)$$

01. $\dfrac{1}{2} + \dfrac{1^2+2^2}{3} + \dfrac{1^2+2^2+3^2}{4} + \ldots..$

2 , 3 , 4 ,.......

$a + (n-1)d$

$2 + (n-1)1$

$2 + n - 1$

$n + 1$

$$tn = \frac{1^2+2^2+3^2+\ldots..+n^2}{n+1}$$

$$= \frac{\dfrac{n(n+1)(2n+1)}{6}}{n+1}$$

$$= \frac{n(2n+1)}{6}$$

$$= \sum_{r=1}^{n} \frac{r(2r+1)}{6}$$

$$= \sum \frac{2r^2+r}{6}$$

$$= \frac{1}{6}\left(2\sum r^2 + \sum r\right)$$

$$= \frac{1}{6}\left(\frac{2\,n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

$$= \frac{1}{6}\left(\frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)}{6}\left(\frac{2n+1}{3} + \frac{1}{2}\right)$$

$$= \frac{n(n+1)}{6}\;\frac{4n+2+3}{6}$$

$$= \frac{n(n+1)(4n+5)}{36}$$

02. $\dfrac{1^2}{1} + \dfrac{1^2+2^2}{1+2} + \dfrac{1^2+2^2+3^2}{1+2+3} + \ldots..$

$$tn = \frac{1^2+2^2+3^2+\ldots..+n^2}{1+2+3+\ldots...+n}$$

$$= \frac{\dfrac{n(n+1)(2n+1)}{6}}{\dfrac{n(n+1)}{2}}$$

$$= \frac{2n+1}{3}$$

$$= \sum_{r=1}^{n} \frac{2r+1}{3}$$

$$= \frac{1}{3}\left(2\sum r + \sum 1\right)$$

$$= \frac{1}{3}\left(2\,\frac{n(n+1)}{2} + n\right)$$

$$= \frac{1}{3}\left(n(n+1) + n\right)$$

$$= \frac{n}{3}\,(n+1+1)$$

$$= \frac{n}{3}\,(n+2)$$

$$= \frac{n(n+2)}{3}$$

03. $\displaystyle\sum \frac{1^2+2^2+3^2+\ldots\ldots+r^2}{r+1}$

$$= \sum \frac{\dfrac{r(r+1)(2r+1)}{6}}{r+1}$$

$$= \sum_{r=1}^{n} \frac{r(2r+1)}{6}$$

$$= \sum \frac{2r^2 + r}{6}$$

$$= \frac{1}{6}\left(2\sum r^2 + \sum r\right)$$

$$= \frac{1}{6}\left(\frac{2\, n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

$$= \frac{1}{6}\left(\frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)}{6}\left(\frac{2n+1}{3} + \frac{1}{2}\right)$$

$$= \frac{n(n+1)}{6} \quad \frac{4n+2+3}{6}$$

$$= \frac{n(n+1)(4n+5)}{36}$$

04. $\sum \dfrac{1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + r^3}{r(r+1)}$

$$= \sum \frac{\frac{r^2(r+1)^2}{4}}{r(r+1)}$$

$$= \sum_{r=1}^{n} \frac{r(r+1)}{4}$$

$$= \sum \frac{r^2 + r}{4}$$

$$= \frac{1}{4}\left(\sum r^2 + \sum r\right)$$

$$= \frac{1}{4}\left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{2n+1}{3} + 1\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{2n+1+3}{3}\right)$$

$$= \frac{n(n+1)(2n+4)}{24}$$

$$= \frac{n(n+1)(n+2)}{12}$$

05. $\sum \dfrac{1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + r^3}{(r+1)}$

$$= \sum \frac{\frac{r^2(r+1)^2}{4}}{(r+1)}$$

$$= \sum_{r=1}^{n} \frac{r^2(r+1)}{4}$$

$$= \sum \frac{r^3 + r^2}{4}$$

$$= \frac{1}{4}\left(\sum r^3 + \sum r^2\right)$$

$$= \frac{1}{4}\left(\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{n(n+1)}{2} + \frac{2n+1}{3}\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{n^2+n}{2} + \frac{2n+1}{3}\right)$$

$$= \frac{n(n+1)}{8}\left(\frac{3n^2 + 3n + 4n + 2}{6}\right)$$

$$= \frac{n(n+1)(3n^2 + 7n + 2)}{48}$$

$$= \frac{n(n+1)(3n^2 + 6n + n + 2)}{48}$$

$$= \frac{n(n+1)\left[3n(n+2) + 1(n+2)\right]}{48}$$

$$= \frac{n(n+1)(n+2)(3n+1)}{48}$$

SOLUTION - QSET 3

01.

$(40^2 - 39^2) + (38^2 - 37^2) + (36^2 - 35^2) + \ldots + (2^2 - 1^2)$

$(2^2 - 1^2) + (4^2 - 3^2) + (6^2 - 5^2) + \ldots + (40^2 - 39^2)$

$t_n = (\ )^2 - (\ )^2$

2 , 4 , 6	1 , 3 , 5 .....
$a + (n-1)d$	$a + (n-1)d$
$2 + (n-1)2$	$1 + (n-1)2$
$2 + 2n - 2$	$1 + 2n - 2$
$2n$	$2n - 1$

$t_n = (2n)^2 - (2n-1)^2$

$$\sum_{r=1}^{20} (2r)^2 - (2r-1)^2$$

$= \sum 4r^2 - (4r^2 - 4r + 1)$

$= \sum 4r^2 - 4r^2 + 4r - 1$

$= \sum 4r - 1$

$= 4\sum r - \sum 1$

$= \dfrac{4n(n+1)}{2} - n$

$= 2n(n+1) - n$

$= 2(20)(21) - 20$

$= 840 - 20$

$= 820$

02.

$(60^2 - 59^2) + (58^2 - 57^2) + (56^2 - 55^2) + \ldots + (2^2 - 1^2)$

REFER ABOVE SOLUTION , BUT PUT n = 30

ans : 1830

04. $31^2 + 32^2 + 33^2 + \ldots\ldots\ldots + 50^2$

$t_n = (\ )^2$

31 , 32 , 33 , ... $a + (n-1)d$

$31 + (n-1)1$

$n + 30$

$$\sum_{r=1}^{20} (r+30)^2$$

$= \sum (r^2 + 60r + 900)$

$= \sum r^2 + 60\sum r + 900\sum 1$

$= \dfrac{n(n+1)(2n+1)}{6} + \dfrac{60n(n+1)}{2} + 900n$

$= \dfrac{n(n+1)(2n+1)}{6} + 30n(n+1) + 900n$

Put n = 20

$= \dfrac{20(21)(41)}{6} + 30(20)(21) + 900(20)$

$= 2870 + 12600 + 18000$

$= 33470$

SOLUTION - QSET 4

01.

$\dfrac{1 + 3 + 5 + 7 + \ldots\ldots \text{ n terms}}{2 + 4 + 6 + 8 + \ldots\ldots \text{ n terms}} = 0.95$

1 , 3 , 5	2 , 4 , 6
$a + (n-1)d$	$a + (n-1)d$
$1 + (n-1)2$	$2 + (n-1)2$
$1 + 2n - 2$	$2 + 2n - 2$
$2n - 1$	$2n$

$\dfrac{\sum (2r-1)}{\sum (2r)} = \dfrac{95}{100}$

$\dfrac{2\sum r - \sum 1}{2\sum r} = \dfrac{19}{20}$

$\dfrac{\dfrac{2n(n+1)}{2} - n}{\dfrac{2n(n+1)}{2}} = \dfrac{19}{20}$

$\dfrac{n(n+1) - n}{n(n+1)} = \dfrac{19}{20}$

$$\frac{n(n + 1 - 1)}{n(n + 1)} = \frac{19}{20}$$

$$\frac{n}{n + 1} = \frac{19}{20}$$

$$20n = 19n + 19$$

$$n = 19$$

02. $$\frac{1 + 2 + 3 + 4 + \dots n \text{ terms}}{1.2 + 2.3 + 3.4 + \dots n \text{ terms}} = 0.03$$

$\boxed{\begin{array}{ll} 1.2 + 2.3 + 3.4 + \dots n \text{ terms} & \\ tn = (\ ).(\ ) & \\ & \\ 1, 2, 3, \dots & 2, 3, 4, \dots \\ a + (n-1)d & a + (n-1)d \\ 1 + (n-1)1 & 2 + (n-1)1 \\ 1 + n - 1 & 2 + n - 1 \\ n & n + 1 \\ & \\ tn = (n)(n+1) & \end{array}}$

$$\frac{\sum r}{\sum r(r + 1)} = \frac{3}{100}$$

$$\frac{\sum r}{\sum (r^2 + r)} = \frac{3}{100}$$

$$\frac{\sum r}{\sum r^2 + \sum r} = \frac{3}{100}$$

$$\frac{\frac{n(n+1)}{2}}{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}} = \frac{3}{100}$$

$$\cancel{\frac{\frac{n(n+1)}{2}}{\frac{n(n+1)}{2}}}\left(\frac{2n+1}{3} + 1\right) = \frac{3}{100}$$

$$\frac{1}{\frac{2n+4}{3}} = \frac{3}{100}$$

$$\frac{3}{2n + 4} = \frac{3}{100}$$

$$2n + 4 = 100$$

$$2n = 96 \qquad n = 48$$

03. $$\frac{1^3 + 2^3 + 3^3 + \dots\dots \ n \text{ terms}}{1.3 + 2.5 + 3.7 + \dots\dots \ n \text{ terms}} = \frac{9}{5}$$

$\boxed{\begin{array}{ll} 1.3 + 2.5 + 3.7 + \dots n \text{ terms} & \\ tn = (\ ).(\ ) & \\ & \\ 1, 2, 3, \dots & 3, 5, 7, \dots \\ a + (n-1)d & a + (n-1)d \\ 1 + (n-1)1 & 3 + (n-1)2 \\ 1 + n - 1 & 3 + 2n - 2 \\ n & 2n + 1 \\ & \\ tn = (n)(2n+1) & \end{array}}$

$$\frac{\sum r^3}{\sum r(2r + 1)} = \frac{9}{5}$$

$$\frac{\sum r^3}{\sum (2r^2 + r)} = \frac{9}{5}$$

$$\frac{\sum r^3}{2\sum r^2 + \sum r} = \frac{9}{5}$$

$$\frac{\frac{n^2(n+1)^2}{4}}{\frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\frac{\frac{n^2(n+1)^2}{4}}{n(n+1)\left(\frac{2n+1}{3} + \frac{1}{2}\right)} = \frac{9}{5}$$

$$\frac{\frac{n(n+1)}{4}}{\frac{2n+1}{3} + \frac{1}{2}} = \frac{9}{5}$$

$$\frac{\frac{n^2+n}{4}}{\frac{4n+2+3}{6}} = \frac{9}{5}$$

$$\frac{n^2+n}{4n+5} \cdot \frac{6}{4} = \frac{9}{5}$$

$$\frac{n^2+n}{4n+5} = \frac{6}{5}$$

$$5n^2 + 5n = 24n + 30$$

$$5n^2 - 19n - 30 = 0$$

$$5n^2 - 25n + 6n - 30 = 0$$

$$5n(n-5) + 6(n-5) = 0$$

$$(n-5)(5n+6) = 0$$

$$n = 5$$